Formation of spinning black holes: semi-empirical mass-spin formula

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Motivation

- Einstein established his general theory of relativity in 1916 --- Einstein equation --- Poisson equation in very strong gravity
- Immediately after, Schwarzschild obtained its solution for a non-rotating/non-spinning black hole (1917)
- It took another half a century to get a solution for a spinning black hole --- Kerr, 1963
- Spin creates all the complicacy

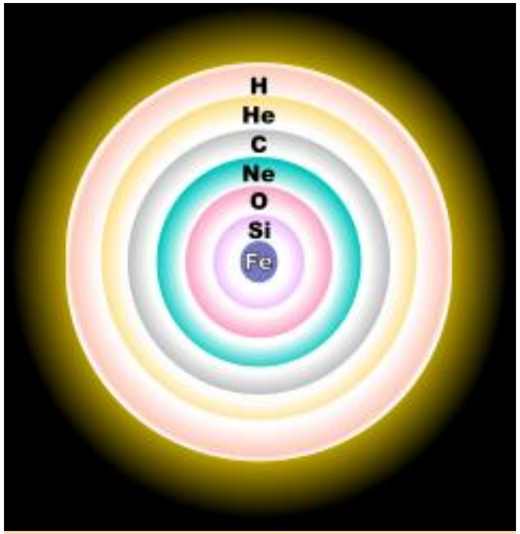
- Observationally, spin of black holes is very difficult to determine due to absence of any hard surface for them --- Questions natural existence of spinning black holes
- Mass determination, however, is easier following, e.g., Newton's law of gravity

Understanding formation history of black holes

- Black holes form from massive stars with mass at least more than 10 M_{Sun} .

- These stars, in particular with $M_{MS} > 20M_{Sun}$, when losing mass rapidly by means of a strong stellar wind, are called **Wolf Rayet stars.**

- When these stars undergo core collapse after losing their hydrogen and/or helium envelope, they form black holes.



The onion-like layers of an evolved massive star
(not to scale)

- If the progenitor star has enough spin, a high density accretion disk develops around the black hole. This is the **Collapsar Model** --- potential model to explain gamma-ray bursts.

- As the core collapses the bulk material of the star explodes into the surrounding ISM leading to a supernova event.

- Observations reveal that many of these supernovae occur in coincidence with the gamma-ray bursts, e.g. GRB 980425 with SN1998bw, GRB 030329 with SN 2003dh etc

COLLAPSAR MODEL : TWO TYPES OF COLLAPSARS

COLLAPSAR TYPE I	COLLAPSAR TYPE II
• Progenitor stars have $M_{MS} > 40\ M_{Sun}$	• Progenitor stars have M_{MS} between $20 - 40\ M_{sun}$
• Form black holes directly during the collapse of the massive core	• Form proto neutron star initially, later transformed to a black hole : "fallback collapsars"
• No supernova explosion :"failed supernovae"	• Mild supernova explosion
• Accretion rate : $0.1 < \dot{M} \leq 10 M_{Sun} s^{-1}$	• Accretion rate is 1-2 orders of magnitude less than that of Type I case .
MacFadyen & Woosley, ApJ 1999.	MacFadyen, Woosley & Heger, ApJ 2001

NEUTRINO-DOMINATED ACCRETION FLOWS

- The mass accretion rates in the collapsar/GRB accretion disks are very high $\sim 10^{-3}$ - 10 M_{sun}/sec .

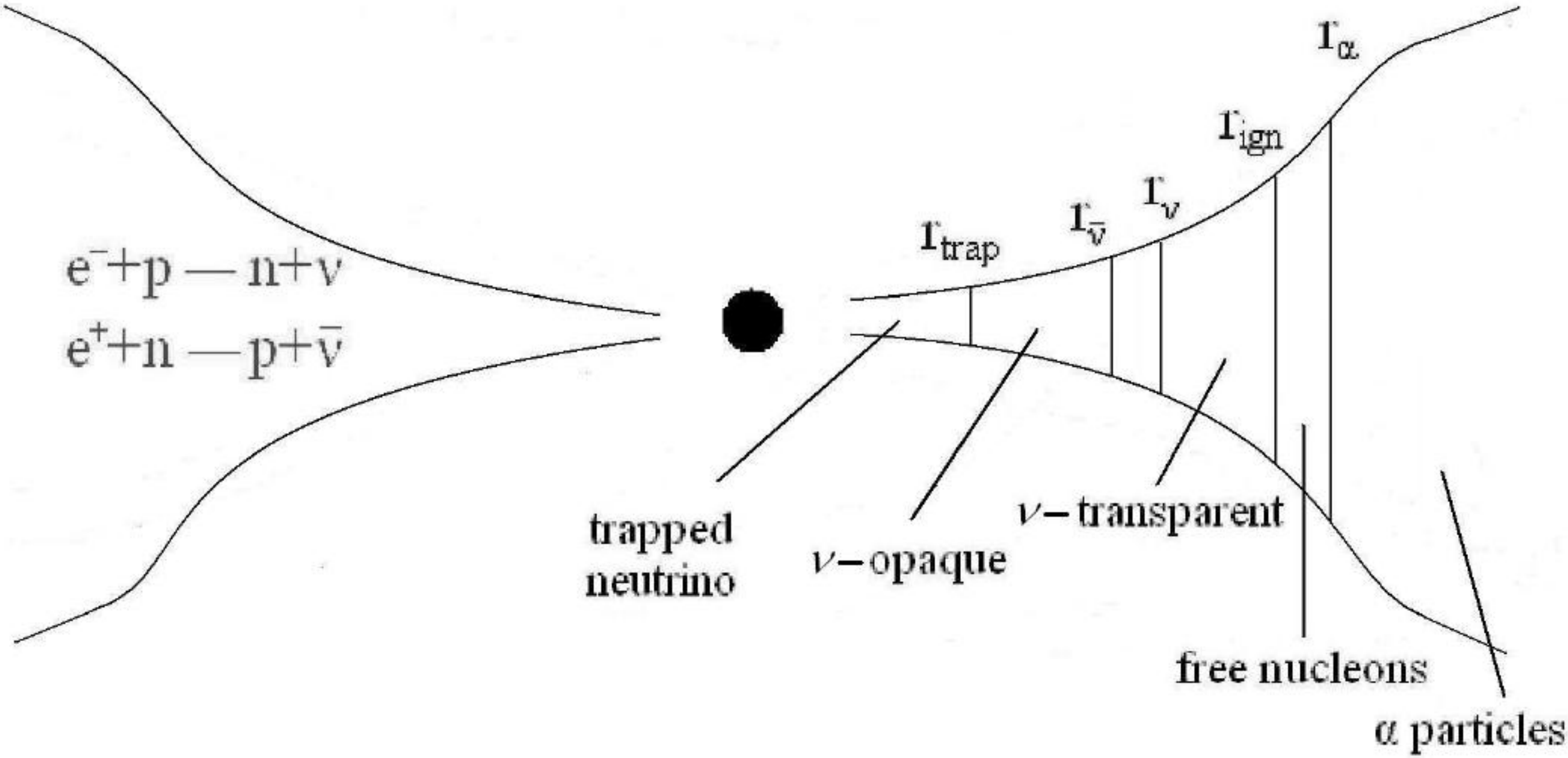
- Disk becomes dense and hot in the inner regions to cool via neutrino emission.

- Neutrinos tap the thermal energy of the disk produced by viscous dissipation and can cool the disk effectively if they manage to escape before being advected into the black hole.

- The outer disk is advection dominated.

Chen & Beloborodov, ApJ 2007; Di Matteo, Perna & Narayan, ApJ 2002

SCHEMATIC PICTURE OF THE DISK WITH CHARACTERISTIC RADII INDICATED



Chen & Beloborodov, ApJ 2007

SIGNIFICANT PROCESSES EMITTING NEUTRINOS

1. **Electron-positron pair annihilation:** $e^- + e^+ \longrightarrow \nu_i + \bar{\nu}_i$

The neutrino cooling rate per unit volume :

$$\bar{q}_{\nu_i,\bar{\nu}_i} \simeq 5 \times 10^{33} T_{11}^9 \text{ ergs cm}^{-3} \text{ s}^{-1}$$

2. **Nucleon–nucleon bremsstrahlung:** $n + n \longrightarrow n + n + \nu_i + \bar{\nu}_i$

The neutrino cooling rate per unit volume :

$$\bar{q}_{\text{brem}} \simeq 10^{27} \rho_{10}^2 T_{11}^{5.5} \text{ ergs cm}^{-3} \text{ s}^{-1}$$

3. **Plasmon decay:** $\tilde{\gamma} \longrightarrow \nu_e + \bar{\nu}_e$

The neutrino cooling rate per unit volume :

$$\bar{q}_{\text{plasmon}} \simeq 1.5 \times 10^{32} T_{11}^9 \gamma_p^6 \exp^{-\gamma_p} (1 + \gamma_p) \left(2 + \frac{\gamma_p^2}{1 + \gamma_p}\right) \text{ ergs cm}^{-3} \text{ s}^{-1}$$

$$\gamma_p = 5.565 \times 10^{-2} [(\pi^2 + \eta_e^2)/3]^{1/2}$$

$$\eta_e = \mu_e / kT$$

4. **Neutronization reactions:** $p + e^- \longrightarrow n + \nu_e$ $\quad n + e^+ \longrightarrow \bar{\nu}_e + p$

The neutrino cooling rate per unit volume:

$$\bar{q}_{eN} = \bar{q}_{e^-p} + \bar{q}_{e^+n} \simeq 9.0 \times 10^{33} \rho_{10} T_{11}^6 X_{\text{nuc}} \text{ ergs cm}^{-3} \text{ s}^{-1}$$

$$X_{\text{nuc}} \approx 34.8 \rho_{10}^{-3/4} T_{11}^{9/8} \exp(-0.61/T_{11})$$

NEUTRINO OPACITIES

- Each neutrino emission process has an inverse process corresponding to absorption:

  The various absorptive optical depths are :

1 . Interaction of neutrinos with one another:

$$\tau_{a,\nu_i\bar{\nu}_i} \approx \frac{q^-_{\nu_i,\bar{\nu}_i} H}{4(7/8)\sigma T^4} \approx 2.5 \times 10^{-7} T_{11}^5 H$$

2 . Absorption onto protons and neutrons:

$$\tau_{a,eN} \approx \frac{q^-_{eN} H}{4(7/8)\sigma T^4} \approx 4.5 \times 10^{-7} T_{11}^2 X_{\rm nuc} \rho_{10} H$$

- Scattering impedes the free escape of neutrinos from the disk.

  Total scattering optical depth is given by:

$$\tau_{s,\nu_i} = 2.7 \times 10^{-7} T_{11}^2 \rho_{10} H$$

DISK MODEL AND INPUT PHYSICS

Height averaged equations describing disk structure:

$$\dot{M} = -2\pi\Sigma\Delta^{1/2}\frac{V}{\sqrt{1-V^2}}$$

$$\frac{V}{1-V^2}\frac{dV}{dr} = \frac{\mathcal{A}}{r} - \frac{1}{\Sigma}\frac{dP}{dr}$$

$$\mathcal{A} = -\frac{M\tilde{A}}{r^3\Delta\Omega_K^+\Omega_K^-}\frac{(\Omega-\Omega_K^+)(\Omega-\Omega_K^-)}{1-\tilde{\Omega}^2\tilde{R}^2},$$

$\tilde{A} = (r^2+a^2)^2 - a^2\Delta\sin^2\theta$, $\Omega = u^\phi/u^t$ is the angular velocity with respect to the stationary observer, $\tilde{\Omega} = \Omega - \omega$ is the angular velocity with respect to the inertial observer, $\Omega_K^\pm = \pm M^{1/2}/(r^{3/2} \pm aM^{1/2})$ are the angular frequencies of the co-rotating and counter-rotating Keplerian orbits, and $\tilde{R} = \tilde{A}/(r^2\Delta^{1/2})$ is the radius of gyration.

$$\frac{\dot{M}}{2\pi}(\mathcal{L}-\mathcal{L}_{in}) = \frac{\tilde{A}^{1/2}\Delta^{1/2}\gamma}{r}\alpha\Pi$$

where $\mathcal{L} = u_\phi$ is the specific angular momentum, γ is the Lorentz factor, $\Pi = 2HP$

$$\frac{\Pi}{\Sigma H^2} = \frac{\mathcal{L}^2 - a^2(\mathcal{E}^2-1)}{2r^4}$$

$$-\frac{\alpha\Pi\tilde{A}\gamma^2}{r^3}\frac{d\Omega}{dr} - \frac{32}{3}\frac{\sigma T^4}{\kappa\Sigma} = -\frac{\dot{M}}{2\pi r\rho}\frac{1}{\Gamma_3-1}\left(\frac{\mathrm{d}P}{\mathrm{d}r} - \Gamma_1\frac{P}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)$$

where T is the temperature in the equatorial plane, κ is the mean (frequency-independent) opacity,

$$\Gamma_1 = \beta^* + (4-3\beta^*)(\Gamma_3-1),$$

$$\Gamma_3 = 1 + \frac{(4-3\beta^*)(\gamma_g-1)}{12(1-\beta/\beta_m)(\gamma_g-1)+\beta},$$

$\beta = P_{\rm gas}/(P_{\rm gas}+P_{\rm rad}+P_{\rm mag})$, $\beta_m = P_{\rm gas}/(P_{\rm gas}+P_{\rm mag})$, $\beta^* = \beta(4-\beta_m)/3\beta_m$, and γ_g is the ratio of specific heats of the gas.

Novikov & Thorne 1973; Abramowicz & Fragile, LRR 2013

Keplerian Solution (V ~ 0, dP/dr ~ 0)

$$F = [7\times 10^{26}\ \mathrm{erg\ cm^{-2}\ s^{-1}}](m^{-1}\dot m)\, r_*^{-3}\, \mathcal{B}^{-1}\mathcal{C}^{-1/2}\mathcal{Q},$$
$$\Sigma = [9\times 10^{4}\ \mathrm{g\ cm^{-2}}](\alpha^{-4/5} m^{1/5}\dot m^{3/5})\, r_*^{-3/5}\, \mathcal{B}^{-4/5}\mathcal{C}^{1/2}\mathcal{D}^{-4/5}\mathcal{Q}^{3/5},$$
$$H = [1\times 10^{3}\ \mathrm{cm}](\alpha^{-1/10} m^{9/10}\dot m^{1/5})\, r_*^{21/20}\, \mathcal{A}\mathcal{B}^{-6/5}\mathcal{C}^{1/2}\mathcal{D}^{-3/5}\mathcal{E}^{-1/2}\mathcal{Q}^{1/5},$$
$$\rho_0 = [4\times 10^{1}\ \mathrm{g\ cm^{-3}}](\alpha^{-7/10} m^{-7/10}\dot m^{2/5})\, r_*^{-33/20}\, \mathcal{A}^{-1}\mathcal{B}^{3/5}\mathcal{D}^{-1/5}\mathcal{E}^{1/2}\mathcal{Q}^{2/5},$$
$$T = [7\times 10^{8}\ \mathrm{K}](\alpha^{-1/5} m^{-1/5}\dot m^{2/5})\, r_*^{-9/10}\, \mathcal{B}^{-2/5}\mathcal{D}^{-1/5}\mathcal{Q}^{2/5},$$
$$\beta/(1-\beta) = [7\times 10^{-3}](\alpha^{-1/10} m^{-1/10}\dot m^{-4/5})\, r_*^{21/20}\, \mathcal{A}^{-1}\mathcal{B}^{9/5}\mathcal{D}^{2/5}\mathcal{E}^{1/2}\mathcal{Q}^{-4/5},$$
$$\tau_{ff}/\tau_{es} = [2\times 10^{-6}](\dot m^{-1})\, r_*^{3/2}\, \mathcal{A}^{-1}\mathcal{B}^{2}\mathcal{D}^{1/2}\mathcal{E}^{1/2}\mathcal{Q}^{-1},$$

The radial functions $\mathcal{A},...,\mathcal{Q}$ that appear in Eqs. (97), (98), (99), are defined in terms of $y = (r/M)^{1/2}$ and $a_* = a/M$ as [237]:

$$\mathcal{A} = 1 + a_*^2 y^{-4} + 2a_*^2 y^{-6}, \qquad \mathcal{B} = 1 + a_* y^{-3},$$
$$\mathcal{C} = 1 - 3y^{-2} + 2a_* y^{-3}, \qquad \mathcal{D} = 1 - 2y^{-2} + a_*^2 y^{-4},$$
$$\mathcal{E} = 1 + 4a_*^2 y^{-4} - 4a_*^2 y^{-6} + 3a_*^4 y^{-8} \qquad \mathcal{Q}_0 = \frac{1 + a_* y^{-3}}{y(1 - 3y^{-2} + 2a_* y^{-3})^{1/2}},$$

$$\mathcal{Q} = \mathcal{Q}_0 \left[y - y_0 - \frac{3}{2}a_* \ln\left(\frac{y}{y_0}\right) - \frac{3(y_1 - a_*)^2}{y_1(y_1 - y_2)(y_1 - y_3)} \ln\left(\frac{y - y_1}{y_0 - y_1}\right)\right]$$
$$- \mathcal{Q}_0 \left[\frac{3(y_2 - a_*)^2}{y_2(y_2 - y_1)(y_2 - y_3)} \ln\left(\frac{y - y_2}{y_0 - y_2}\right) - \frac{3(y_3 - a_*)^2}{y_3(y_3 - y_1)(y_3 - y_2)} \ln\left(\frac{y - y_3}{y_0 - y_3}\right)\right]$$

Here $y_0 = (r_{\rm ms}/M)^{1/2}$, and y_1, y_2, and y_3 are the three roots of $y^3 - 3y + 2a_* = 0$; that is

$$y_1 = 2\cos[(\cos^{-1} a_* - \pi)/3],$$
$$y_2 = 2\cos[(\cos^{-1} a_* + \pi)/3],$$
$$y_3 = -2\cos[(\cos^{-1} a_*)/3].$$

For the numerical solutions reproduced in Figure 7, the opacities were assumed to be $\kappa_{es} = 0.34\ \mathrm{cm^2\ g^{-1}}$ and $\kappa_{ff} = 6.4\times 10^{22}\rho_{\rm cgs} T_{\rm K}^{-7/2}\ \mathrm{cm^2\ g^{-1}}$, where $\rho_{\rm cgs}$ is the density in g cm^{-3} and $T_{\rm K}$ is the temperature in Kelvin.

Solution depends on spins of accretion disk and black hole which is going through the formation stage

Entire process takes about few 10s of sec and finally a fully developed black hole forms

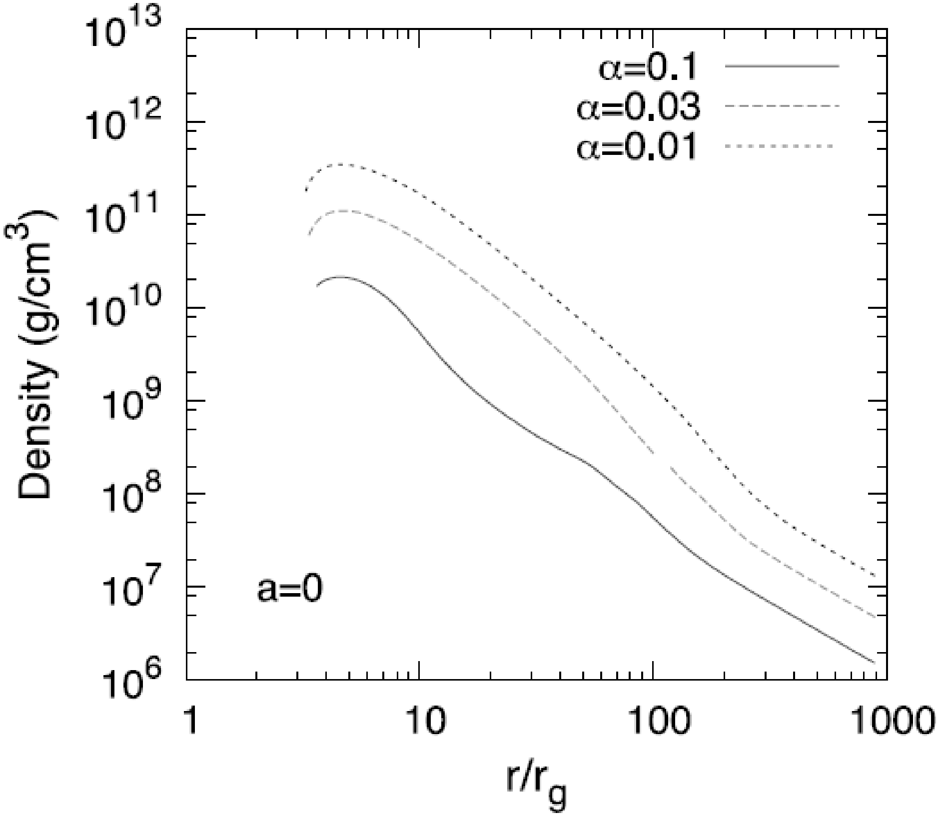
↓

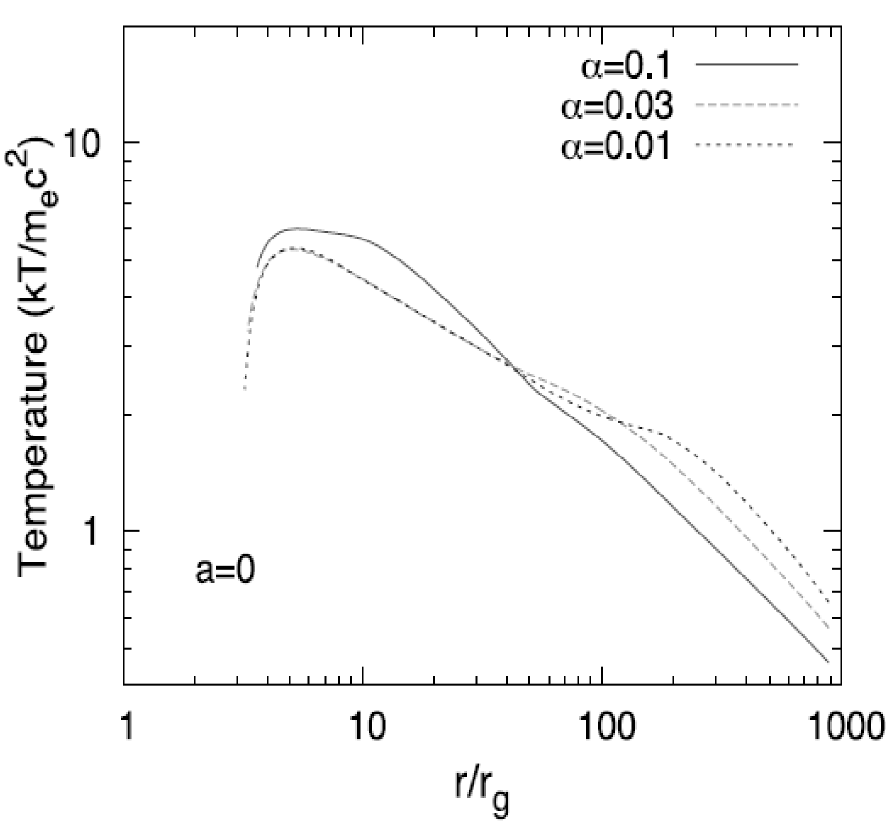
A rotating massive star could give rise to a rotating black hole

Novikov & Thorne 1973; Abramowicz & Fragile, LRR 2013

DENSITY AND TEMPERATURE PROFILES

Collapsar I





$M_{BH} = 3M_{Sun}$

$\dot{M}$ = 0.2 M_{Sun}/sec

POSSIBLE WAY OF BLACK HOLE SPIN MEASUREMENT: BASED ON KNOWN CORRELATION

- Until last decade, there was no estimate of spin of known stellar mass black holes

- Even today various models predict different spins for the known black holes

If the radius of the event horizon, R_+, is known, then specific angular momentum, a, and mass, M, of black holes can be correlated:

$$R_+ = M + (M^2 - a^2)^{1/2}$$

- Usually, R_+ is predicted from observation of various black hole sources

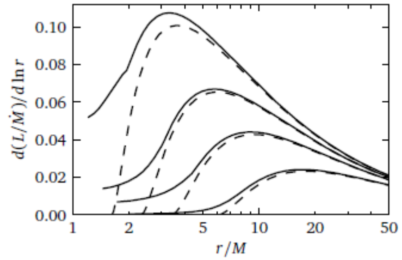
- Mass, M, is obtained from an independent measurement

- Thus, spin can be determined

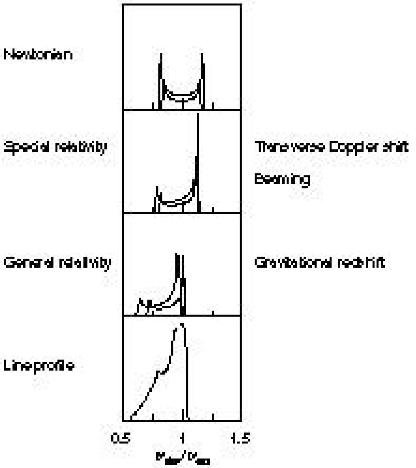
- However, we cannot fix R_+ uniquely for all black holes → Event horizon does not serve any generic purpose to correlate mass with spin

Methods to measure spin: floating around

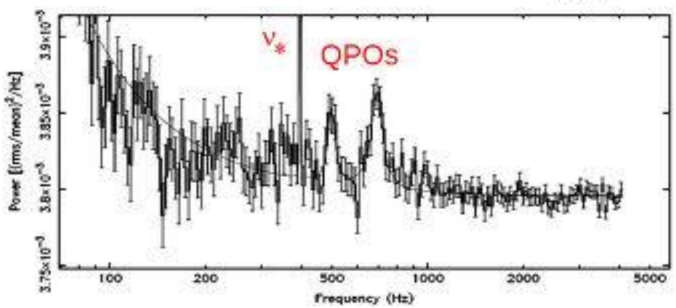
- Continuum fitting:
  Harvard Group



- Iron-line:
  Cambridge-Maryland Group



- Quasi-Periodic Oscillation:
  IISc Group



❖ Different methods/groups produce different results for same black holes: e.g. GRS 1915+105 --- rapidly spinning according to Continuum fitting, but moderately spinning for other methods

Proposal of a fundamentally different approach to verify/predict the spin

- Probe the formation stage/evolution history of black holes: Collapsar accretion disk

- Model the neutrino flux of collapsar disks which is determined by mass, spin and accretion rate of the underlying black hole
- Neutrino luminosity lies in a narrow range:

  Collapsar I → ~ 10^{52} erg/sec → Optically thick flow

  Collapsar II → ~ 10^{49} erg/sec → Optically thin flow
- For a given mass and Collapsar model (fixing accretion rate), spin of the black hole can be determined
- A formula correlating mass, spin and accretion rate is proposed

BASIC IDEA
COLLAPSAR

Due to neutrino emission, disk is cooled: behaves like a Keplerian disk

↓

Inner region of collapsar I disks mimics the celebrated Shakura-Sunyaev/Novikov-Thorne (1973) model

↓

The various flow variables in the Shakura-Sunyaev/Novikov-Thorne model are expressed in terms of explicit algebraic formulae

↓

Neutrino luminosity from the disk is roughly constant ~ $10^{52} – 10^{53}$ erg/s.

↓

The flux is ~ $10^{36} – 10^{37}$ erg/cm^2/s at R ~ 10M

For collapsar II disk, flow is advection dominated with lower luminosity and flux: Flow is sub-Keplerian

Collapsar I & II

↓

For known luminosity, flux is modeled from collapsar accretion disk

Semi-Empirical Formula

Model accretion disk in general relativity

Flux (of neutrino) coming out of the disk::

$$F = [7 \times 10^{26} \text{ erg cm}^2 \text{ s}^{-1}](\dot{m}\, m^{-1}) r_*^{-3} B^{-1} C^{-1/2} Q$$

$$B = 1 + a_* y^{-3},$$

$$C = 1 - 3y^{-2} + 2a_* y^{-3},$$

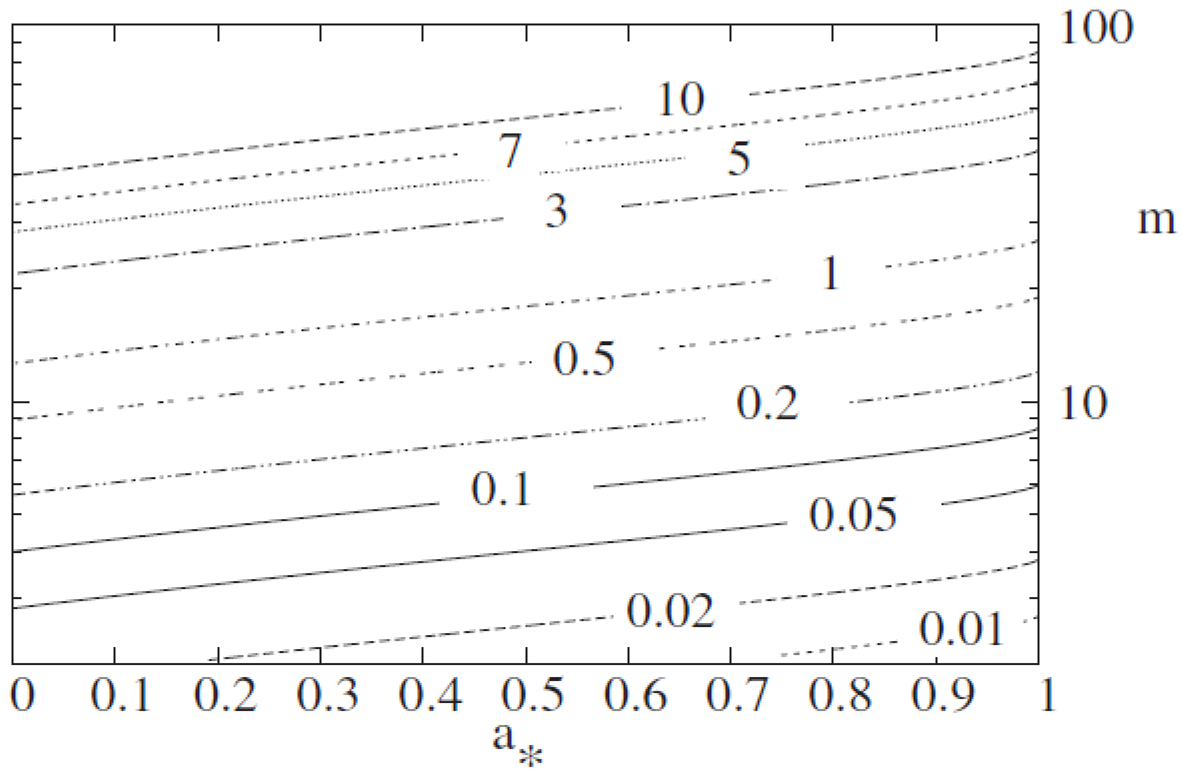
Observed luminosity and hence flux in the inner zone of the disk lies in a narrow range which fixes LHS → Various combinations of mass, spin and accretion rate produce same flux → establishing our semi-empirical formula

Banerjee & Mukhopadhyay, PRL 2013

MASS variation with KERR PARAMETER

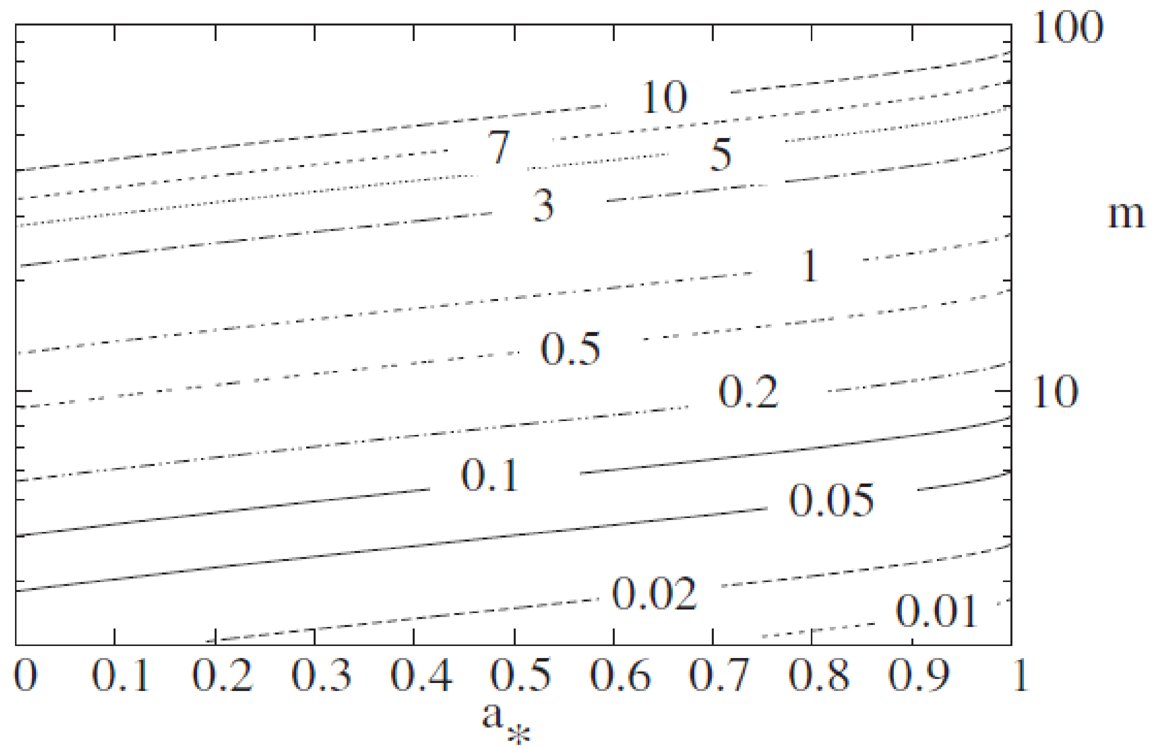
In the Collapsar I Scenario

We fix the flux at R ~ 10M, vary accretion rate from 0.01-10 M_{Sun}/sec, a* from 0-1 and obtain the mass accordingly.



Banerjee & Mukhopadhyay, PRL 2013

Spins are natal→ will change beyond the life time of Universe



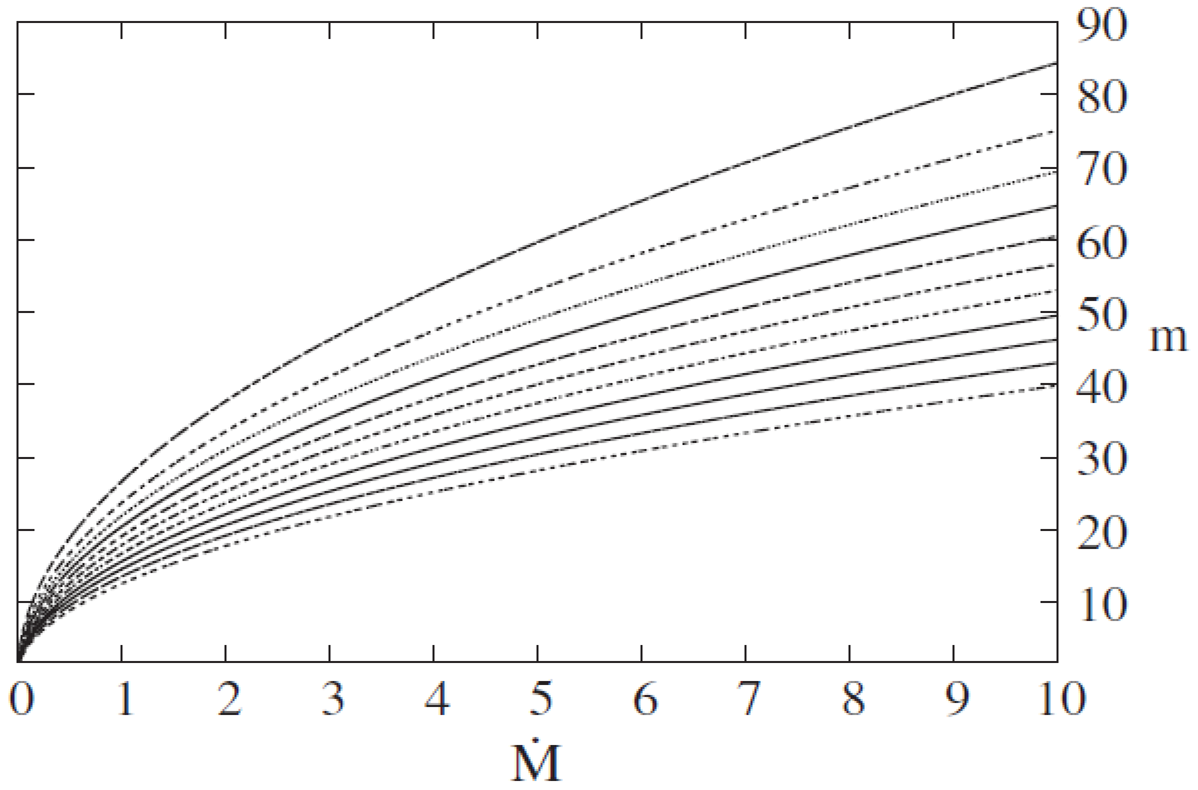
- **Maximum mass of stellar mass black hole ~ 85 M_{sun} (from our calculations)**

- **Maximum possible mass of stellar mass black hole ranges: ~ 10M_{sun} (for super-solar metallicity) to ~ 80 M_{sun} (for extremely low metallicity)**

Belczynski et al., ApJ 2010

- **Our theory predicts that such massive black holes are maximally spinning and their formation time accretion rate is ~ 10 M_{Sun}/sec.**

MASS Variation with ACCRETION RATE

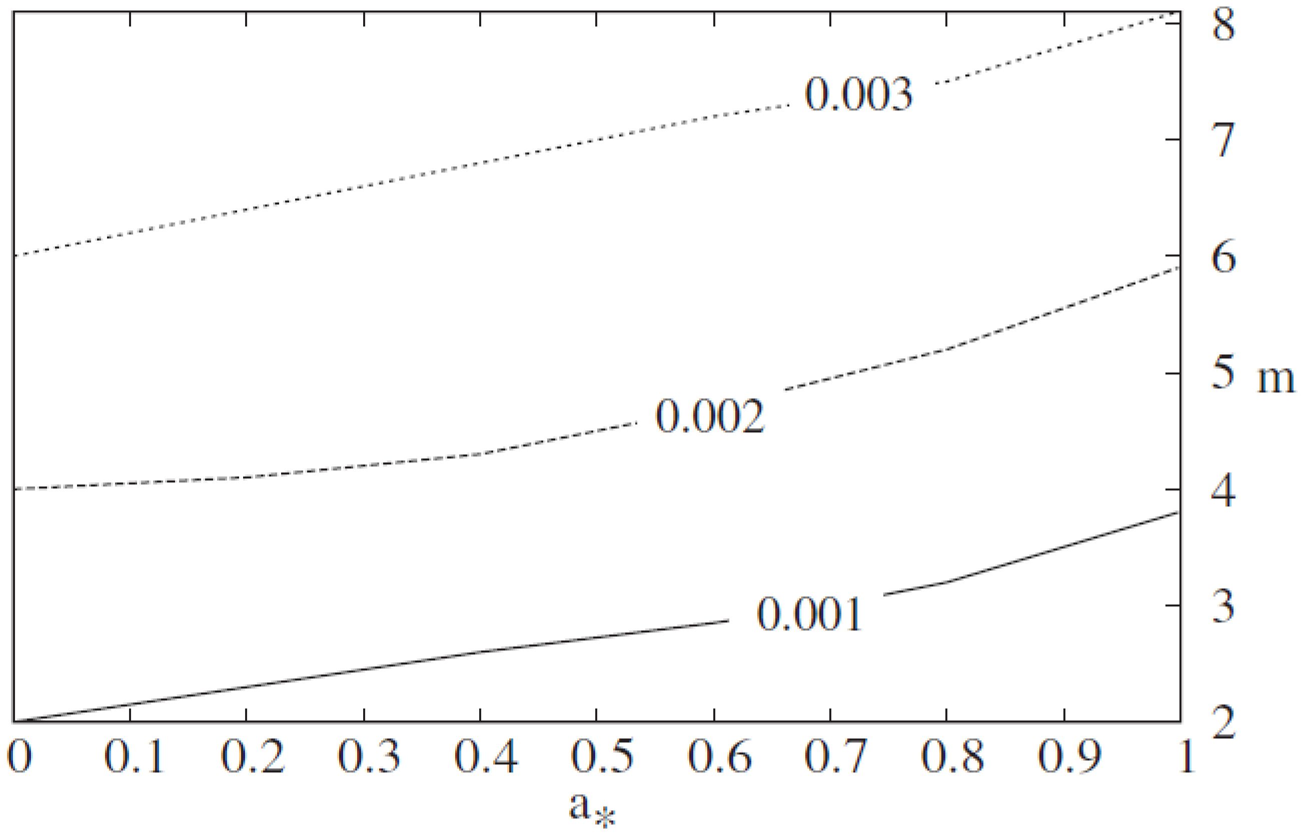


In the Collapsar I Scenario

Banerjee & Mukhopadhyay, PRL 2013

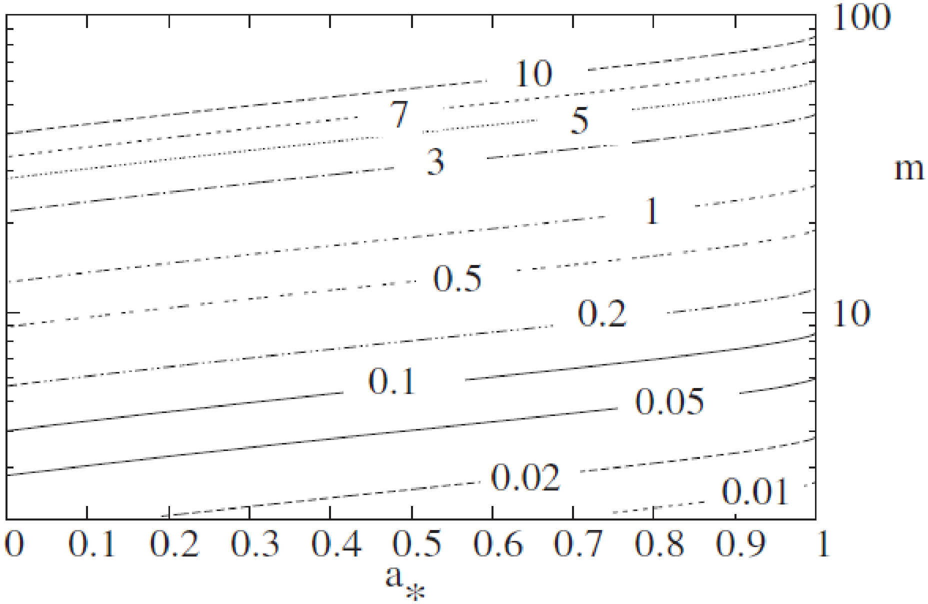
MASS variation with KERR PARAMETER

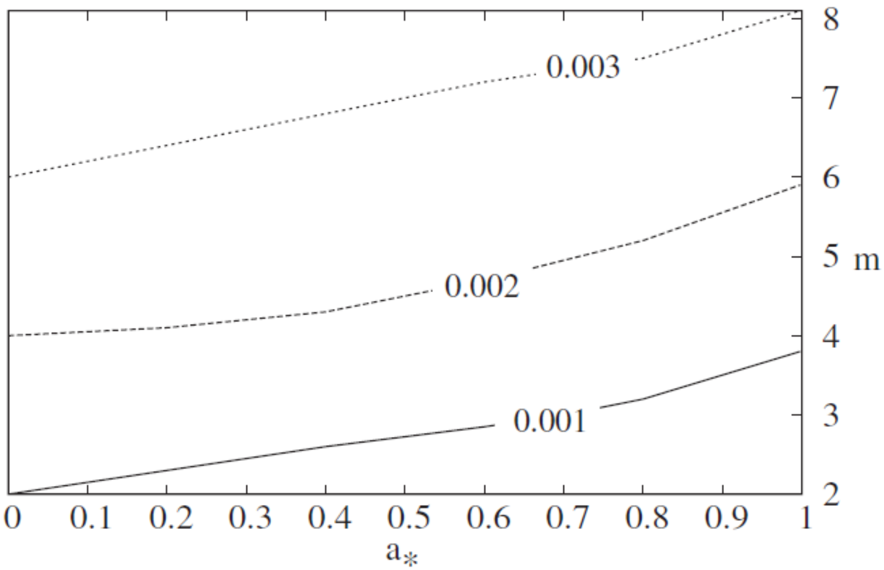
In the Collapsar II Scenario



Banerjee & Mukhopadhyay, PRL 2013

IMPORTANCE OF ACCRETION RATE IN CORRELATING MASS WITH SPIN





Mass and spin of known BHs (referred from existing literature)

BH Candidate	a_*	$M(M_\odot)$
A0620 – 00	0.12 ± 0.19 [4]	6.61 ± 0.25
XTE J1550 – 564	0.34 ± 0.24 [4] 0.7 ± 0.01 [22]	9.10 ± 0.61
GRO J1655 – 40	0.7 ± 0.1 [4] 0.75 ± 0.01 [22]	6.30 ± 0.27
GRS 1915 + 105	0.975 ± 0.025 [4] 0.68 ± 0.08 [22]	14.0 ± 4.4
4U 1543 – 47	0.8 ± 0.1 [4]	9.4 ± 1.0

Spin for Supermassive Black Holes

- Mass-spin formula can be applied for supermassive black holes --- because the sources have definite luminosity/flux
- We choose a few Quasars having Keplerian disk close to the black hole with blackbody radiation: the scaled up Collapsar I model

  We searched for 80 black holes:

  Some sample results ----
- PG 1322-659 (radio quiet): 0.937 to 0.999
- PG 1704+608 (radio loud): 0.887
- PG 0947+396 (radio quiet but jet): 0.582
- PG 0049+171 (radio quiet): 0.276
- PG 0050+124 (radio quiet): -0.815 to -1

Banerjee & Mukhopadhyay (in preparation)

Summary

- Discussed formation criteria of spinning black holes
- Propose a Semi-Empirical Mass-Spin Formula → Mass and spin should not be considered independent parameters
- Mass of black holes is easier to determine by an independent method → Spin measurement is a challenge
- Using our formula, spin can be predicted for a known mass
- We predict that maximum mass of stellar mass black holes can be as high as $85M_{Sun}$ which tallies with that inferred based on observed data. Our theory establishes that these black holes are maximally spinning
- Primarily the spin is that at the formation stage of black hole
- King & Kolb (MNRAS 1999), McClintock et al. (ApJ 2006) and Lee et al. (ApJ 2002) showed that the spin of black holes that we observe today is chiefly natal: mass and spin of black hole hardly evolve during accretion → the mass-spin relation remains true today