1 Models of Dark Energy

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Summary. In this talk we present a pedagogical review of scalar field dynamics. The main emphasis is put on the underlying basic features rather than on concrete scalar field models. Cosmological dynamics of standard scalar fields, phantoms and tachyon fields is developed in detail. Scaling solutions are discussed emphasizing their importance in modelling dark energy. The developed concepts are implemented in an example of *quintessential* inflation. A brief discussion of scaling solutions for coupled quintessence is also included.

Accelerated expansion seems to have played an important role in the dynamical history of our universe. There is a firm belief, at present, that universe has passed through inflationary phase at early times and there have been growing evidences that it is accelerating at present. The recent measurement of the Wilkinson Microwave Anisotropy Probe (WMAP) in the Cosmic Microwave Background (CMB) made it clear that (i) the current state of the universe is very close to a critical and that (ii) primordial density perturbations that seeded large-scale structure in the universe are nearly scale-invariant and Gaussian, which are consistent with the inflationary paradigm. As for the current accelerating of universe, it is supported by observations of high redshift type Ia supernovae treated as standardized candles and, more indirectly, by observations of the cosmic microwave background and galaxy clustering. The criticality of universe supported by CMB observations fixes the total energy budget of universe. The study of large scale structure reveals that nearly 30 percent of the total cosmic budget is contributed by dark matter. Then there is a deficit of almost 70 percent; the supernovae observations tell us that the missing component is an exotic form of energy with large negative pressure dubbed *dark energy*[1, 2, 3, 4]. The recent observations on baryon oscillations provides yet another independent support to dark energy hypothesis. The idea that universe is in the state of acceleration is slowly establishing in modern cosmology.

The dynamics of our universe is described by Einstein equations in which the contribution of energy content of universe is represented by energy momentum tensor appearing on RHS of these equations. The LHS represents pure geometry given by the curvature of space time. Einstein equations in their original form with energy momentum tensor of normal matter can not lead to acceleration. There are then two ways to obtain accelerated expansion, either by supplementing energy momentum tensor by dark energy component or by modifying the geometry itself. In the frame work of Dvali-Gabadadze-Porrati (DGP) brane worlds[5], the extra dimensional effects can lead to late time acceleration. The other alternative which is largely motivated by phenomenological considerations is related to the introduction of inverse powers of Ricci scalar in the Einstein Hilbert action[6]. The third intriguing possibility is provided by Bekenstein relativistic theory of modified gravity[7, 8, 9] which apart from spin two field contains a vector and a scalar field.

Due to the simplicity of the mechanism, most of the work in cosmology related late time acceleration is attributed to the assumption that within the framework of general relativity, cosmic acceleration is sourced by an energy-momentum tensor which has a large negative pressure. The simplest candidate of dark, yet most difficult from field theoretic point of view, is provided by cosmological constant. Due to its non evolving nature it is plagued with fine tuning problem which can be alleviate in dynamically evolving scalar field models. A variety of scalar field models have been conjectured for this purpose including quintessence [10, 11], phantoms[12, 13, 14], K-essence [15] and recently tachyonic scalar fields[16]. In this talk we present a review of cosmological dynamics of quintessence, phantoms and rolling tachyons. We describe in detail the concepts of field dynamics relevant to cosmic evolution with a special emphasis on scaling solutions. The example of quintessential inflation is worked out in detail.

We employ the metric signature (-,+,+,+) and use the reduced Planck mass $M_p^{-2} = 8\pi G \equiv \kappa^2$. In certain places we have adopted the unit $M_p = 1$. Finally we should mention that our list of references is restricted, in most of the places, we referred to reviews to help the readers.

1.1 Glimpses of FRW cosmology

The Freidmann-Robertson-Walker (FRW) model is based on the assumption of homogeneity and isotropy which is approximately true at very large scales. The small deviation from homogeneity at early epochs played very important role in the dynamical history of our universe. The small density perturbations are believed to have grown via gravitational instability into the structure we see today in the universe. the origin of primordial perturbations is quantum mechanical and is out side the scope of standard big bang model. In what follows we shall review main features of FRW model necessary for the subsequent sections.

Homogeneity and isotropy forces the metric of space time to assume the form[17]

$$
\begin{aligned}
& ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta \mathrm{d}\phi^2)\right) , \\
& k = 0, \pm 1 ,
\end{aligned}
\tag{1.1}
$$

where $a(t)$ is cosmic scale factor. Coordinates r, θ and ϕ are known as *comoving* coordinates. A freely moving particle comes to rest in these coordinates.

Eq.(1.1) is purely a kinematic statement. In this problem the dynamics is associated with the scale factor− a(t). Einstein equations allow to determine the scale factor provided the matter content of universe is specified. Constant k occurring in the metric (1.1) describes the geometry of spatial section of space-time. Its value is also determined once the matter distribution in the universe is known. Observations have repeatedly confirmed the spatially flat geometry ($k = 0$) in confirmation of the prediction of inflationary scenario.

1.1.1 Evolution equations

The differential equations for the scale factor and the matter density follow from Einstein equations

$$G^{\mu}_{\nu} \equiv R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu} \,, \tag{1.2}$$

where$G_{\mu,\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor which depends on the metric and its derivatives and R is Ricci scalar. The energy momentum tensor $T_{\mu\nu}$ assumes a simplified form reminiscent of ideal perfect fluid in FRW background

$$T^{\mu}_{\nu} = Diag\,(-\rho, p, p, p) \ . \tag{1.3}$$

In this case the components of $G_{\mu\nu}$ can easily be computed

$$G^0_0 = -\frac{3}{a^2}\left(\dot{a}^2 + k\right) \tag{1.4}$$

$$G^j_i = \frac{1}{a^2}\left(2a\ddot{a} + \dot{a}^2 + k\right) \tag{1.5}$$

and all the other components of Einstein tensor are identically zero. Equations (1.2) then give the two independent equations

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} - \frac{k}{a^2} \tag{1.6}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) \ . \tag{1.7}$$

The energy momentum tensor is conserved by virtue of the Bianchi identity $\Delta^{\nu} G^{\mu}_{\nu} = 0$ leading to the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0 \,. \tag{1.8}$$

Equations (1.6), (1.7) & (1.8) make a redundant set of equations convenient to use; one of the two equations (1.7) & (1.8) can be obtained using the other one and the Hubble equation (1.6). These equations supplemented with the equation of state $p(t) = p(\rho)$ uniquely determine $a(t)$, $p(t)$ and $\rho(t)$. Constant k also gets determined

$$\frac{k}{a^2} = H^2 \left(\Omega(t) - 1 \right) , \tag{1.9}$$

where $\Omega = \rho/\rho_c$ is the dimensionless density parameter and $\rho_c = 3H^2/8\pi G$ is critical density. The matter distribution clearly determines the spatial geometry of our universe namely

$$\Omega < 1 \quad or\ \rho < \rho_c \quad \rightarrow k = -1 \tag{1.10}$$

$$\Omega = 1 \quad or\ \rho = \rho_c \quad \rightarrow k = 0 \tag{1.11}$$

$$\Omega > 1 \quad or\ \rho > \rho_c \quad \rightarrow k = 1 \ . \tag{1.12}$$

In case of $k = 0$, the value the scale factor at the present epoch a_0 can be normalized to a convenient value, say, $a_0 = 1$. In other cases it should be determined using the observed values of H_0 and $\Omega^{(0)}$ from the relation $a_0 H_0 = \left(|\Omega^{(0)} - 1| \right)^{-1/2}$. Observations on cosmic micro wave background radiation support the *critical* universe which is one of the predictions of inflation. We would therefore assume $k = 0$ in the subsequent description.

- **Acceleration**

We now turn to the nature of expansion which is determined by the matter content in the universe. Eq.(1.7) should be contrasted to the analogous situation in Newtonian gravity

$$\ddot{R} = -\frac{4\pi}{3} G \rho R \tag{1.13}$$

where R denotes the distance of the test particle from the center of a homogeneous sphere of mass density ρ. In general theory of relativity (GR), unlike the Newtonian case, pressure contributes to energy density and may qualitatively modify the dynamics. Indeed, from Eq.(1.7) we have

$$\ddot{a} > 0 \qquad if\ p < -\frac{\rho}{3} \tag{1.14}$$

$$\ddot{a} < 0 \qquad if\ p > -\frac{\rho}{3} \ . \tag{1.15}$$

Accelerated expansion, thus, is fuelled by an exotic form of matter of large negative pressure dubbed *dark energy* which turns gravity into a repulsive force. The simplest example of a perfect fluid of negative pressure is provided by cosmological constant associated with $\rho = constant$. In this case the continuity equation (1.8) yields the relation $p = -\rho$. A host of scalar field systems can also mimic negative pressure.

Assuming that the universe is filled with perfect barotropic fluid with constant equation of state parameter $w = p/\rho$ yields

$$\rho \propto a^{-3(1+w)} \tag{1.16}$$

$$a(t) \propto t^{\frac{2}{3(1+w)}} \quad (w > -1) \tag{1.17}$$

$$a(t) \propto e^{H\ t} \qquad (w = -1) \ . \tag{1.18}$$

The last equation corresponds to cosmological constant which can be added to the energy momentum tensor of the perfect fluid. Interestingly, in four dimension and at the classical level, the only modification Einstein equations allow is associated with $T_{\mu\nu} \to +T_{\mu\nu} + \Lambda g_{\mu\nu}$. Historically such a modification was first proposed by Einstein to achieve a static solution which turns out to be unstable. It was later dropped by him after the Hubble's discovery. In presence of Λ, the evolution equations modify to

$$H^2 = \frac{8\pi G}{3} + \frac{\Lambda}{3}\,, \tag{1.19}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\,(\rho + 3p) + \frac{\Lambda}{3}\ . \tag{1.20}$$

From Eq.(1.20), it clearly follows that Λ term contributes negatively to the pressure term and hence exhibits repulsive effect.

• **Age crisis and cosmological constant**

Apart from the dark energy problem, cosmological constant has other important implications, in particular, in relation to the age problem. In any cosmological model with normal form of matter, the age of universe falls short as compared to the age of some well known old objects found in the universe. Remarkably, the presence of Λ can resolve the age problem. In order to appreciate the problem, let us first consider the case of flat dust dominated universe ($\Omega_m = 1$)

$$a(t) \propto t^{2/3} \to \quad H_0 = \frac{2}{3t_0}\ . \tag{1.21}$$

The present value of the Hubble parameter H_0 is not accurately know by the observations

$$H_0^{-1} = h^{-1} 0.98 \times 10^{10} years\,, \tag{1.22}$$

$$0.8 < h < 0.64 \to \quad to = (8 - 10) \times 10^9 years. \tag{1.23}$$

This model is certainly in trouble as its prediction for age of universe fails to meet the solar age constraint $-$ $t_0 > (11 - 12) \times 10^9 years$. One could try to improve the situation by invoking the open model with $\Omega_m^{(0)} < 1$. In this case the age of universe is expected to be larger than the flat dust dominated model$-$ for less amount of matter, it would take longer for gravitational interaction to slow down the expansion rate to its present value. Indeed, in this case we have the exact expression

$$H_0 t_0 = \frac{1}{1 - \Omega_m^{(0)}} - \frac{\Omega_m^{(0)}}{2(1 - \Omega_m^{(0)})^{3/2}} \cosh^{-1}\left(\frac{2 - \Omega_m^{(0)}}{\Omega_m^{(0)}}\right) \tag{1.24}$$

from which follows that

$$H_0 t_0 = 1\,, for \quad \Omega_m^{(0)} \to 0\,, \tag{1.25}$$

$$H_0 t_0 = \frac{2}{3}\,, for \quad \Omega_m^{(0)} \to 1\ . \tag{1.26}$$

For obvious reasons, in case of the closed universe, the age would even be smaller than $2/3H_0^{-1}$. Though the age of universe is larger than $2/3H_0^{-1}$ for vanishingly small value of $\Omega_m^{(0)}$, such a model is note viable as $\Omega_m^{(0)} \simeq 0.3$ and universe is critical to a good accuracy. The problem can be solved in a flat universe dominated by cosmological constant. In fact, in a flat universe with two components ($\Omega_m^{(0)} + \Omega_\Lambda^{(0)} = 1$), the Hubble equation

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\Omega_m^{(0)} \left(\frac{a_0}{a}\right)^2 + \Omega_\Lambda^{(0)}\right] \tag{1.27}$$

has the solution

$$\frac{a}{a_0} = \left(\frac{\Omega_m^{(0)}}{\Omega_\Lambda^{(0)}}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2}\Omega_m^{(0)\,1/2} H_0 t\right) , \tag{1.28}$$

which at $t = t_0$ yields the following expression for the age of universe

$$t_0 = \frac{2}{3}\frac{H_0^{-1}}{\Omega_\Lambda^{(0)\,1/2}} \ln\left(\frac{1 + \Omega_\Lambda^{(0)\,1/2}}{\Omega_m^{(0)\,1/2}}\right) . \tag{1.29}$$

In Fig.1.1, we have plotted the age t_0 versus Ω_m. The age of universe is larger than H_0^{-1} for a Λ dominated universe. The numerical value of t_0 ($t_0 \simeq 0.96H_0^{-1}$) is comfortable with observations for popular values of $\Omega_m^{(0)} = 0.3$ and $\Omega_\Lambda^{(0)} = 0.7$.

• **Super acceleration**

So far, we have restricted our attention to fluids with equation' of state parameter $w \geq -1$. The case of $w < -1$ corresponds to *phantom dark energy* and requires separate considerations. The power law expansion $a(t) \sim t^n (n = 2/3(1+w))$ corresponds to shrinking universe for $n < 0$ ($w < -1$). The situation can easily be remedied by changing the sign of t and by introducing the origin of time t_s

$$a(t) = (t_s - t)^n , \tag{1.30}$$

which is the generic solution of evolution equations for super-negative values of w and it gives rise to a very different future course of evolution

$$H = \frac{n}{t_s - t} \tag{1.31}$$

$$R = 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] = 6\frac{n(n-1) + n^2}{(t_s - t)^2} . \tag{1.32}$$

The Hubble expansion rate diverges as $t \to t_s$ corresponding to infinitely large energy density after a finite time in future. The curvature also grows to infinity as $t \to t_s$. Such a situation is referred to Big Rip singularity. Big Rip

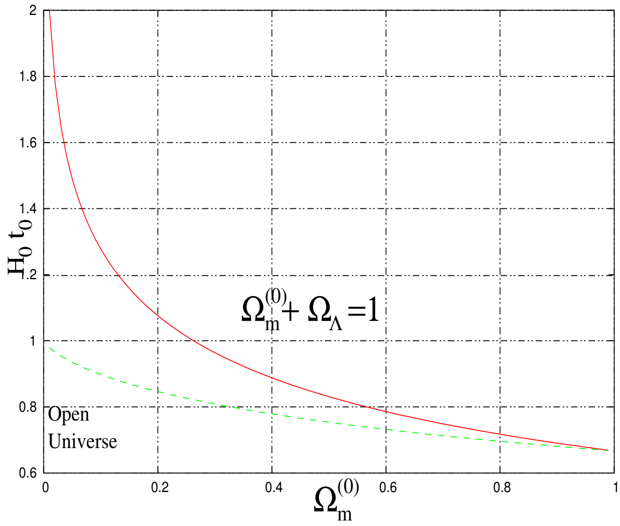

Fig. 1.1. Age of universe (in the units of H_0^{-1}) is plotted against $\Omega_m^{(0)}$ in a flat model (red line) with $\Omega_m^{(0)} + \Omega_\Lambda^{(0)} = 1$ and matter dominated model (green line) with $\Omega_m^{(0)} = 1$.

can be avoided in specific models of phantom field with variable equation of state. It should also be emphasized that quantum effects become important in a situation when curvature becomes large. In that case one should take into account the higher order curvature corrections to Einstein Hilbert action which crucially modifies the structure of singularity.

1.1.2 Scalar fields– as perfect fluids in FRW background

Scalar fields naturally arise in unified models of interactions and also in string theory. Since the invent of inflation, they continue play an important role in cosmology. They are frequently used as candidates of dark energy. In the recent years a variety of scalar field models namely quintessence, phantoms, tachyons, K-essence, dilatonic ghosts and others have been investigated in the literature. In what follows we briefly describe some of these systems. Their dynamics will be dealt with in detail in section III.

• **Standard scalar field**

Let us consider the scalar field minimally coupled to gravity

$$S = -\int \left(\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right)\sqrt{-g}d^4x \ . \qquad (1.33)$$

The Euler Lagrangian equation

$$\partial^{\mu}\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\partial^{\mu}\phi}-\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\phi}=0\,, \tag{1.34}$$
$$\sqrt{-g}=a^{3}(t)$$

for the action (1.33) in case of a homogeneous field acquires the form

$$\ddot{\phi}+3H\dot{\phi}+V_{\phi}=0\,, \tag{1.35}$$

which is equivalent to the conservation equation

$$\frac{\dot{\rho}}{\rho}+3H(1+w)=0\ . \tag{1.36}$$

The energy momentum tensor

$$T_{\mu\nu}=-2\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \tag{1.37}$$

for the field ϕ which arises from the action (1.33) is given by

$$T_{\mu\nu}=\partial_{\mu}\phi\partial_{\nu}\phi-g_{\mu\nu}\left[\frac{1}{2}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi+V(\phi)\right]\ . \tag{1.38}$$

In the homogeneous and isotropic universe, the field energy density ρ_{ϕ} and pressure p_{ϕ} obtained from $T_{\mu\nu}$ are

$$T_{00}\equiv\rho=\frac{\dot{\phi}^{2}}{2}+V(\phi),\qquad T_{i}^{i}\equiv p=\frac{\dot{\phi}^{2}}{2}-V(\phi)\ . \tag{1.39}$$

The field evolution equation
(1.35) formally integrates to

$$\rho=\rho_{0}e^{-6\int\left(1-\frac{2V}{\dot{\phi}^{2}+2V}\right)\frac{da}{a}}\ . \tag{1.40}$$

Thus the scaling of field energy density crucially depends upon the ratio of kinetic to potential energy. Depending upon the scalar field regime ρ can mimic a behavior ranging from cosmological constant to stiff matter

$$\rho\sim a^{-m}\qquad 0<m<6\ . \tag{1.41}$$

This behavior is also clear intuitively namely the field ϕ rolling slowly along the flat wing of the potential gives rise to $p\simeq-\rho$ where as it gives $p\simeq\rho$ while dropping fast along the steep part of the potential. Interestingly, one can obtain the similar picture in the oscillatory regime for a power law type of potential.

• **Acceleration during oscillations**.
As the scalar field evolves towards the minimum of its, the slow role ceases and a the scalar field enters into the regime of quasi periodic evolution with

decaying amplitude. In what follows , we shall assume that the potential is even and has minimum at $\phi = 0$. When the field initially being displaced from the minimum of the potential, rolls below its slow roll value, the coherence oscillation regime, $\nu >> H$, commences. The evolution equation can then be approximately solved by separating the two times scales namely the fast oscillation time scale and the longer expansion time scale. On the first time scale, the Hubble expansion can be neglected, and one obtain ϕ as a function of time;

$$t - t_0 = \pm \int \frac{1}{\sqrt{2(V_m - V(\phi))}} d\phi \ , \tag{1.42}$$

where $\rho \equiv V_m \equiv V(\phi_m)$; V_m being the maximum current value of the potential energy and ϕ_m being the field amplitude . On the longer time scale ρ and ϕ_m slowly decrease because Hubble damping term in equation (1). The average adiabatic index γ is defined as

$$\gamma = \left\langle \frac{\rho + p}{\rho} \right\rangle = \left\langle \frac{\dot{\phi}^2}{\rho} \right\rangle \ , \tag{1.43}$$

where$< . >$ denotes the time average over one oscillation . Equation (4) then tells that expansion during oscillations would continue ($\ddot{a} > 0$) if $\gamma < \frac{2}{3}$. The adiabatic evolution of $a(t)$ and ρ is given by ,

$$a(t) \propto t^{\frac{2}{3\gamma}} \ , \tag{1.44}$$

$$\rho \equiv V(\phi_m) \propto t^{-2} \ . \tag{1.45}$$

As $\ddot{\phi} = \frac{-dV}{d\phi}$, the condition $\gamma < \frac{2}{3}$ can equivalently be written

$$\gamma = \left\langle \frac{\dot{\phi}^2}{\rho} \right\rangle = \frac{< \phi V_{,\phi} >}{V_m} = 2(1 - \frac{< V >}{V_m}) \ ,$$

$$= 2 \frac{\int_0^{\phi_m} (1 - V(\phi)/V_m)^{\frac{1}{2}} d\phi}{\int_0^{\phi_m} (1 - V(\phi)/V_m)^{-\frac{1}{2}} d\phi} = \frac{2p}{p+1} \ , \tag{1.46}$$

for a power law potential $V \sim \phi^{2p}$, which gives the average value of the equation of state parameter

$$\langle w \rangle = \frac{p-1}{p+1}, \qquad p < 1/2 \ \rightarrow \ acceleration \, . \tag{1.47}$$

Thus a quadratic potential, on the average, mimics dust where as the quartic potential exhibits radiation like behavior. It is really interesting that the scalar field in oscillatory regime can give rise to dark energy for $p < 1/2$.

While developing scalar fields models of dark energy, it is important to have some control on its dynamics. In what follows we show how to construct a field potential viable to desired cosmic evolution.

• Construction of field potential for a given cosmological evolution

After the invent of cosmological inflation, scalar field models have been frequently used in cosmology in various contexts; they play a central role specially in modelling dark energy. Our focus in the review will also be around these models. We should, however, caution the reader that the scalar field models have limited predictive power. The merits of these models should therefore be judged on the basis of generic features that might emerge in them. Indeed, for *a priori* given cosmological evolution, we can always construct a field potential that would produce it. We shall illustrate this simple fact in case of a power law expansion for a general cosmological background governed by the Friedmann equation

$$H^2 = \frac{\rho^q}{A} \,, \tag{1.48}$$

where $q = 2, 2/3$ correspond to Randall-Sundrum (RS) and Gauss-Bonnet (GB) brane worlds respectively; A is a constant which takes different values in different patches. We show below how to construct the field potential for ordinary scalar field propagating in a general background described by (1.48).

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} \ . \tag{1.49}$$

Using Eqs. (1.36) and (1.48) we obtain

$$1 + w = -\left(\frac{2}{3q}\right)\frac{\dot{H}}{H^2} \ . \tag{1.50}$$

From evolution equation (1.49) and the expression $\dot{\phi}^2 = V(1+w)(1-w)^{-1}$, we have the differential equation for the field potential V

$$\frac{\dot{V}}{V} = -\frac{\dot{f} + 6Hf}{1+f} \,, \tag{1.51}$$

where $f = (1+w)(1-w)^{-1}$. Integrating (1.51) respecting (1.50), we get

$$V(t) = \frac{C}{3q}\left(\frac{3qH^2 + \dot{H}}{H^{2(q-1)/q}}\right) \,, \tag{1.52}$$

where $C = A^{1/q}$ is an integrating constant. Expressing f in terms of H and its derivative through equation (1.50) and using (1.51), we obtain the $\phi(t)$

$$\phi(t) = \left(\frac{2C}{3q}\right)^{1/2} \int \left[-\frac{\dot{H}}{H^{2(q-1)/q}}\right]^{1/2} dt \ . \tag{1.53}$$

Equations (refpoteq) and (1.53) allow to find the field potential with a given expansion dynamics prescribed by $a(t)$. For $a(t) \sim t^n$, we are interested, we have

$$\phi - \phi_0 = D_{nq} t^{(q-1)/q} \ , \tag{1.54}$$

$$V(t) = C n^{2/q} \left(1 - \frac{n^{-(q+2)/2q}}{3q}\right) t^{-2/q} \ , \tag{1.55}$$

where $D_{nq} = n^{(2-q)/q}(2C/3q)^{1/2}(q/q-1)$ and $q \neq 1$. Combining (1.54) and (1.55) we get the expression for the potential as function of ϕ

$$V(\phi) = V_0 \phi^{-(2/q-1)} \ . \tag{1.56}$$

In case $q = 1$, the field logarithmically depends on time t and Eq. (1.56) leads to the well known exponential potential. For $q = 2$ corresponding to RS, we obtain $V(\phi) \sim 1/\phi^2$. The case of high energy GB regime ($q = 2/3$) leads to the power law behavior of $V(\phi)$

$$V(\phi) = V_0 \phi^6 \ . \tag{1.57}$$

• **Phantom Field**.

All these models of scalar field lead to the equation of state parameter w greater than or equal to minus one. However, the recent observations do not seem to exclude values of this parameter less than minus one. It is therefore important to look for theoretical possibilities to describe dark energy with $w < -1$ called phantom energy. In our opinion, the simplest alternative is provided by a phantom field, scalar field with negative kinetic energy. Such a field can be motivated from S-brane constructs in string theory. Historically, phantom fields were first introduced in Hoyle's version of the Steady State Theory. In adherence to the Perfect Cosmological Principle, a creation field (C-field) was for the first time introduced [12] to reconcile with homogeneous density by creation of new matter in the voids caused by the expansion of Universe. It was further refined and reformulated in the Hoyle and Narlikar theory of gravitation [13]. Though the quantum theory of phantom fields is problematic, it is nevertheless interesting to examine the cosmological consequences of these fields at the classical level.

The Lagrangian of the phantom field minimally coupled to gravity is given by

$$\mathcal{L} = (16\pi G)^{-1} R + \frac{1}{2} g^{\mu\nu} \partial\phi_\mu \partial\phi_\nu - V(\phi) \ , \tag{1.58}$$

where $V(\phi)$ is the phantom potential. The kinetic energy term of the phantom field in (1.58) enters with the opposite sign in contrast to the ordinary scalar field (we remind the reader that we use the metric signature, -,+,+,+). In a spatially flat FRW cosmology, the stress tensor that follows from (1.58) acquires the diagonal form $T^\alpha_\beta = diag\,(-\rho, p, p, p)$ where the pressure and energy density of field ϕ are given by

$$\rho = -\frac{\dot{\phi}^2}{2} + V(\phi), \qquad p = -\frac{\dot{\phi}^2}{2} - V(\phi) \ . \tag{1.59}$$

The corresponding equation of state parameter is now given by

$$w \equiv \frac{p}{\rho} = \frac{\frac{\dot{\phi}^2}{2} + V(\phi)}{\frac{\dot{\phi}^2}{2} - V(\phi)} \ . \tag{1.60}$$

For $\rho > 0$, $w < -1$.

The equations of motion which follow from (1.58) are

$$\dot{H} = \frac{1}{2M_p^2}\dot{\phi}^2 \tag{1.61}$$

$$H^2 = \frac{1}{3M_p^2}\rho_\phi \tag{1.62}$$

$$\ddot{\phi} + 3H\dot{\phi} = V'(\phi) \ . \tag{1.63}$$

Note that the evolution equation (1.63) for the phantom field is same as that of the normal scalar field with inverted potential allowing the field with zero initial kinetic energy to roll up the hill; i.e., from lower value of potential to higher one. At the first look such a situation looks pathological. However, at present, the situation in cosmology is remarkably tolerant to any pathology if it can lead to a viable model.

As mentioned above the equation of state parameter with super negative values leads to Big Rip which can be avoided in a particular class of models. For instance, let us consider consider a model with

$$V(\phi) = V_0 \left[\cosh\left(\frac{\alpha\phi}{M_p}\right)\right]^{-1} \ . \tag{1.64}$$

Due to its peculiar properties, the phantom field, released at a distance from the origin with zero kinetic energy, moves to wards the top of the potential and crosses over to the other side and turns ba ck to execute the damped oscillation about the maximum of the potential (see Fig.1.2). After a certain period of ti me the motion ceases and the field settles on the top of the potential permanently to mimic the de-Sitter like behavior ($w = -1$).

- **Rolling tachyon**

It was recently suggested that rolling tachyon condensate, in a class of string theories, might have interesting cosmological consequences. It was shown by Sen[16] that the decay of D-branes produces a pressure-less gas with finite energy density that resembles classical dust. Rolling tachyon has an interesting equation of state whose parameter smoothly interpolates between -1 and 0. Attempts have been made to construct viable cosmological model using rolling tachyon field as a suitable candidate for inflaton, dark matter or dark

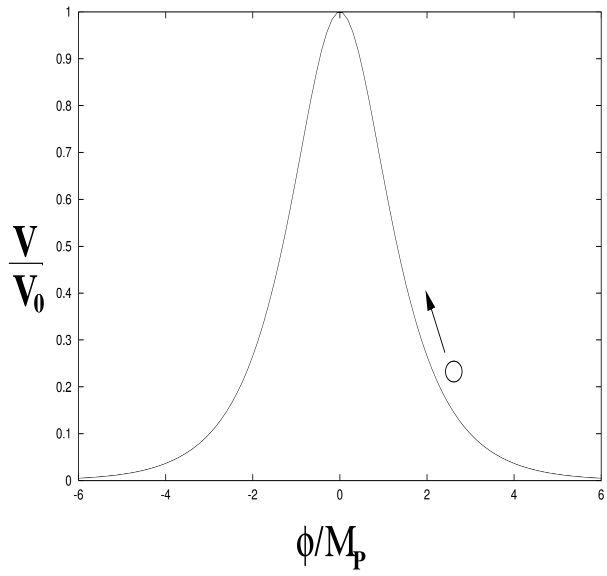

Fig. 1.2. Evolution of the phantom field is shown for the model described by eqn. (1.64). Due to the unusual behavior, the phantom field, released with zero kinetic energy away from the origin, moves towards the top of the potential. It sets into the damped oscillations about $\phi = 0$ and ultimately settles there permanently.

energy (see Ref.[3] and references therein for details). As for the inflation, the rolling tachyon models are faced with difficulties associated with reheating. In what follows we shall consider the tachyon potentials field to obtain viable models of dark energy.

The tachyon dynamics (on a non-BPS) D_3 brane can be described by an effective field theory with the following action

$$S = \int d^4x \Big\{ \sqrt{-g} \left(\frac{R}{2\kappa^2} \right) - V(\phi) \sqrt{-\det(g_{ab} + \partial_a \phi \partial_b \phi)} \Big\} . \tag{1.65}$$

The tachyon field measures the varying brane tension and is such that $V(\phi = \infty) = 0$ and $V(\phi = 0) = 1$. The effective potential obtained in open string theory has the form

$$V(\phi) = \frac{T_3}{\cosh\left(\frac{\phi}{\phi_0}\right)} , \tag{1.66}$$

where $\phi_0 = \sqrt{2}$ for superstring and $\phi_0 = 2$ in case of bosonic string. We should note that the potential for the rolling scalar contains no free parameter to tune which is normally required for a viable cosmological evolution. For instance, the late time evolution of the scalar field with potential (1.66) can mimic the current accelerated expansion of universe provided the brane tension T_3 could be tuned to the critical energy density at the current epoch. However, this is absolutely out of scope from viewpoint of string theory as it leads to very small masses for massive string states. We are, therefore, led to think of another mechanism which would affect the D-brane tension and the slope of the scalar field potential without touching the string length and the string coupling constant. We shall hereafter show that these features are shared by the warped compactification. Consider the following warped metric

$$ds_{10}^2 = \beta(y_i) g_{ab} dx^a dx^b + \beta^{-1}(y_i) \hat{g}_{ij} dy^i dy^j \,, \tag{1.67}$$

where the coordinates y_i represent the compact dimensions, and $\hat{g}_{ij}$ represent metric in the compact space. At some point in the y-space the factor β can be small. This corresponds to a scenario in which the brane moves in the compact dimensions reducing its tension. The tachyon action at a point y in the y-space becomes

$$S = -\int d^4x \beta^2 V(\phi) \sqrt{-\det(g_{ab} + \beta^{-1}\partial_a\phi\partial_b\phi)} \,. \tag{1.68}$$

Normalizing the scalar field as $\phi \to \sqrt{\beta}\phi$, one finds the standard Dirac-Born-Infeld (DBI) type action

$$S = -\int d^4x V(\phi) \sqrt{-\det(g_{ab} + \partial_a\phi\partial_b\phi)} \,, \tag{1.69}$$

where now the potential is

$$V(\phi) = \frac{V_0}{\cosh\left(\frac{\sqrt{\beta}\phi}{\phi_0}\right)} \,, \quad \text{with} \quad V_0 = \beta^2 T_3 \,. \tag{1.70}$$

The constant V_0 can be less than T_3 for small values of β with $\beta < 1$. In sections to follow, we shall also consider other forms of tachyon potential which can be inspired by string theory and others which are introduced by purely phenomenological considerations.

In a spatially flat Friedmann-Robertson-Walker (FRW) background, The energy density ρ and the pressure p which follow from action (1.65) are given by,

$$\rho = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \,, \tag{1.71}$$

$$p = -V(\phi)\sqrt{1-\dot{\phi}^2} \,. \tag{1.72}$$

The equation of motion of the rolling scalar field follows from Eq. (**??**)

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_\phi}{V(\phi)} = 0\,, \tag{1.73}$$

which is equivalent to the conservation equation

$$\frac{\dot{\rho}}{\rho} + 3H(1+w) = 0\ . \tag{1.74}$$

The tachyon dynamics is very different from the standard field case. Irrespective of steepness of tachyon potential, its equation of state parameter varies between 0 and −1. Thus reheating is impossible to achieve in this model, if tachyon field is to be an inflaton. However, it can be used as a candidate of dark energy as shown in one of the following sections.

We now look for the potential which can lead to power law type of expansion in case of tachyon field. In this case, the expression for $(1+w)$ is also given by the Eq. (1.50) but the equation of state parameter w has a simple relation with $\dot{\phi}$

$$\dot{\phi}^2 = 1 + w\ . \tag{1.75}$$

Using (1.50) and (1.75) we get

$$\phi(t) = \int \left[-\frac{2\dot{H}}{3qH^2} \right] dt\ . \tag{1.76}$$

From Eqs. (1.48), (1.50) and (1.72) we can express the potential $V(t)$ as

$$V(t) = (-w)^{1/2}\rho = H^{2/q}A^{1/q}\left(1 + \frac{2}{3q}\frac{\dot{H}}{H^2}\right)\ . \tag{1.77}$$

In case of Born-Infeld scalar field, Eqs. (1.76) and (1.77) determine the field $\phi(t)$ and the potential $V(t)$ for given scale factor $a(t)$. In case of power law expansion $a(t) \propto t^n$, we obtain from (1.76) and (1.77)

$$\phi(t) - \phi_0 = \left(\frac{2}{3nq}\right)^{1/2} t\ , \tag{1.78}$$

$$V(t) = n^{2/q}A^{1/q}\left(1 - \frac{2}{3nq}\right)^{1/2} t^{-2/q}\ , \tag{1.79}$$

which finally lead to

$$V(\phi) = V_0\phi^{-2/q}\ . \tag{1.80}$$

We should note that the power law expansion in the present case takes place with the constant velocity of the the field (see Eq. (1.78)) which is typical of Born-Infeld dynamics. For $q = 1$ corresponding to standard GR, (1.80) reduces to inverse square potential earlier obtained by Padmanabhan[18]. In case of RS which corresponds to $q = 2$, we get $V(\phi) \sim 1/\phi$. In case of high energy GB regime (= 2/3), the potential which can implement power law expansion turns out to be

$$V(\phi) = \frac{V_0}{\phi^3} \ . \tag{1.81}$$

This sort of hierarchy of potentials is understandable; in GR the required potential behaves as $1/\phi^2$ whereas in RS scenario due to the extra brane damping, the power law expansion can be achieved with the less inverse power of field . In the high energy GB regime, the Hubble damping is weaker than the standard FRW cosmology, thereby requiring larger inverse power of the field.

1.1.3 Current acceleration and observations in brief

The direct evidence of current acceleration of universe is related to the observation of luminosity distance of high redshift supernovae by two groups independently in 1998[1, 2]. The luminosity distance at high redshift is larger in dark energy dominated universe. Thus supernovae would appear fainter in case the universe is dominated by dark energy. The luminosity distance can be used to estimate the apparent magnitude m of the source given its absolute magnitude M. using the following relation often used in astronomy

$$m - M = 5\log\left(\frac{D_L}{Mpc}\right) + 25 \ . \tag{1.82}$$

In order to get a feeling of the phenomenon (the reader is referred to excellent review of Perivolaropoulos[19] for details) let us consider two supernovae 1997ap at redshift $z = 0.83$ with $m = 24.32$ and 1992P at $z = 0.026$ with apparent magnitude $M = 16.08$. Since the supernovae are supposed to be the standard candles, their absolute magnitude is same. Secondly we shall use the fact that $D_L(z) \simeq z/H_0$ for small value of z. Eq.(1.82) then yields the following estimate

$$H_0 D_L \simeq 1.16 \tag{1.83}$$

The theoretical estimate for the luminosity distance for flat universe tells us

$$D_L \simeq 0.95 H_0^{-1}, \quad \Omega_m^{(0)} = 1\,, \tag{1.84}$$

$$D_L \simeq 1.23 H_0^{-1}, \quad \Omega_m^{(0)} = 0.3,\ \Omega_\Lambda^{(0)} = 0.7 \ . \tag{1.85}$$

The above estimate clearly lands a strong support to the case of dark energy dominated universe (see Ref.[19] for details).

An interesting proposal for visualizing acceleration in supernovae data was proposed in Ref.[20]. The authors displayed the data with error bars on the phase plane $(\dot{a}, a)$, see Fig.1.3 for flat models with different values of Ω_m. The data at low red shift clearly confirms the presence of accelerated phase but due to large error bars it is not possible to choose a particular model.

The later requires the interplay between the low redshift and the high redshift data[20]. The observations related to CMB and large scale structures independently support the dark energy scenario. The CMB anisotropies observed originally by COBE in 1992 and the recent WMAP data overwhelmingly support inflationary scenario. The location of the major peak around $l = 220$ tells us that $\Omega_{tot} \simeq 1$. Since the baryonic matter in the universe amounts to only 4%. Nearly 30% of the total energy content is contributed by non-luminous component of non-barionic nature with dust like equation of state dubbed *cold dark matter*. There is then a deficit of about 70 %–the missing component, known as dark energy. The CMB and the large scale galaxy clustering data is complimentary to supernova results; the combined analysis strongly points towards $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ universe.

However, in view of the fine tuning problem, it looks absolutely essential that dark energy be represented by a variable equation of state. At the same time, the quest for dark energy metamorphosis continues at the observational level.

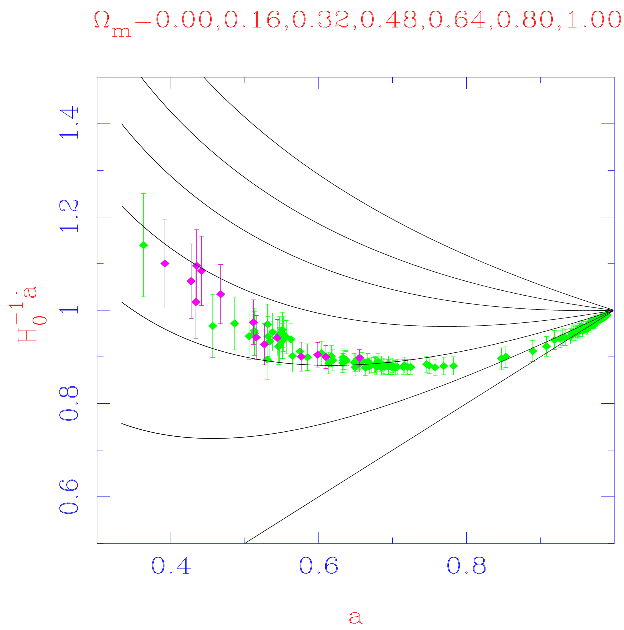

Fig. 1.3. The supernovae data points are displayed in the phase plane $(\dot{a}, a)$. The solid curves correspond to flat cosmological models for different values of Ω_m. The bottom and top curves corresponds to $\Omega_m = 0.0, 1.0$ respectively from[20]

1.2 Cosmological constant Λ

Historically Λ was introduced by Einstein to achieve a static solution which turned out to be unstable. However, after the Hubble's redshift discovery in 1929, the motivation for having Λ was lost and it was dropped. Since then the cosmological constant was introduced time and again to remove the discrepancies between theory and observations and withdrawn when these discrepancies were resolved. It had come and gone several times making its come back finally, seemingly for ever!, in 1998 through supernova Ia observations. Recently much efforts have gone in understanding Λ in the frame work of quantum fields and string theory. In what follows we shall briefly mention these issues.

• **Λ as a natural free parameter of classical gravity**
It should be noted that a term proportional to the the metric $g_{\mu\nu}$ is missing on the right hand side of Einstein equations (1.2). Indeed the Bianch identity $\Delta^\nu G^\mu_\nu = 0$ implies that

$$G_{\mu\nu} = +\kappa T_{\mu\nu} - \Lambda g_{\mu\nu} \ , \tag{1.86}$$

with

$$\nabla_\nu T^{\mu\nu} = 0 \ , \tag{1.87}$$

where $T_{\mu\nu}$ is a symmetric tensor, and κ and Λ are constants. The demand that it should in the first approximation reduce to the Newtonian equation for gravitation will require $T_{\mu\nu}$ to represent the energy momentum tensor for matter and $\kappa = 8\pi G/c^2$ with Λ being negligible at the stellar scale. The Einstein equations should then read as

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \ . \tag{1.88}$$

Note that the constant Λ enters into the equation naturally. It was introduced by Einstein in an ad-hoc manner to have a physically acceptable static model of the Universe and was subsequently withdrawn when Friedmann found the non-static model with acceptable physical properties. We would however like to maintain that it appears in the equation as naturally as the stress tensor $T_{\mu\nu}$ and hence should be considered on the same footing[21]. As for the classical physics, the cosmological constant is a free parameter of the theory and its numerical value should be determined from observations.

• **Λ arising due to vacuum fluctuations**.
Cosmological constant can be associated with vacuum fluctuations in the quantum field theoretic context. Though the arguments are still at the level of numerology but may have far reaching consequences. Unlike the classical theory the cosmological constant Λ in this scheme is no longer a free parameter of the theory. Broadly the line of thinking takes the following route.

The quantum effects in GR become important when the Einstein Hilbert action becomes of the order of Planck's constant; this happens at the Planck's length $L_p = \sqrt{(8\pi G)} \sim 10^{-32} cm$ corresponding to Planck energy which is of the order of $M_p^4 \sim 10^{72} GeV^4$. In the language of field theory, a system is described by a set of quantum fields. The ground state energy dubbed zero point energy or *vacuum* energy of a free quantum field is infinite.

This contribution is related the ordering ambiguity of fields in the classical Lagrangian and disappears when normal ordering is adopted. Since this procedure of throwing out the vacuum energy is ad hoc, one might try to cancel it by introducing the counter terms. The later, however requires fine tuning and may be regarded as unsatisfactory. Whether or not the zero point energy in field theory is realistic is still a debatable question. The divergence is related to the modes of very small wave length. As we are ignorant of physics around Planck scale we might be tempted to introduce a cut off at L_p and associate Λ with this fundamental scale. Thus we arrive at an estimate of vacuum energy $\rho_v \sim M_p^4$ (corresponding mass scale$-$ $M_V \sim (\rho_V^{1/4})$ which is away by 120 orders of magnitudes from the observed value of this quantity. The vacuum energy may not be felt in the laboratory but plays important role in GR through its contribution to the energy momentum tensor as $< T_{\mu\nu} >_0 = -\rho_V g_{\mu\nu}$ and appears on the right hand side of Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left(T_{\mu\nu} + < T_{\mu\nu} >_0 \right) \ . \tag{1.89}$$

The problem of zero point energy is naturally resolved by invoking supersymmetry which has many other remarkable features. In the supersymmetric description, every bosonic degree of freedom has its Fermi counter part which contributes zero point energy with opposite sign compared to the bosonic degree of freedom thereby doing away with the vacuum energy. It is in this sense the supersymmetric theories do not admit a non-zero cosmological constant. However, we know that we do not leave in supersymmetric vacuum state and hence it should be broken. For a viable supersymmetric scenario, for instance if it is to be relevant to hierarchy problem, the suppersymmetry breaking scale should be around $M_{susy} \sim 10^3 GeV$. We are still away from the observed value by many orders of magnitudes. At present we do not know how Planck scale or SUSY breaking scales is related to the observed vacuum scale.

• **Λ from string theory$-$ de-Sitter vacuua *a la* KKLT**

In view of the observations related to supernova, large scale clustering and Micro wave background, the idea of late time acceleration has reached the level of general acceptability. It is, therefore, not surprising that tremendous efforts have recently been made in finding out de-Sitter solutions in supergravity and string theory. Using flux compactification, Kachru, Kallosh, Linde and Trivedi (KKLT) formulated a procedure to construct de-Sitter vacua of type IIB string theory[22]. They demonstrated that the life time of the vacua is larger that the age of universe and hence these solutions can be considered as stable for practical purposes. Although a fine-tuning problem of Λ still remains in this scenario, it is interesting that string theory gives rise to a stable de-Sitter vacua with all moduli fixed. We note that a vast number of different choices of fluxes leads to a complicated landscape with more than 10^{100} vacua. We should believe, if we can, that we live in one of them!. .

1.2.1 Fine tuning problem

Inspite of the fact that introduction of Λ does not require an adhoc assumption and it is also not ruled out by observation as a candidate of dark energy; the scenario base upon Λ is faced with the worst type of fine tuning problem. The numerical value of Λ at early epochs should be tuned to a fantastic accuracy so as not to disturb todays physics. In order to appreciate the problem, let us consider the following ratio

$$\frac{\rho_\Lambda}{\frac{3H^2(t)}{8\pi G}} = \Omega_\Lambda \left(\frac{H_0}{H(t)}\right)^2 , \tag{1.90}$$

where $\Omega_\Lambda = (\rho_\Lambda/\rho_c) \simeq 0.7$. It will not disturb our estimate if we assume radiation domination today. In that case the ratio H/H_0 scales as a^{-2} and since the temperature is inversely proportional to the scale factor a, we find

$$\frac{\rho_\Lambda}{\frac{3H^2(t)}{8\pi G}} = 0.7 \left(\frac{T_0}{T}\right)^4 . \tag{1.91}$$

Since at the Planck ($T = T_p = M_p$) epoch $T_0/T \simeq 10^{-31}$, the ratio of ρ_Λ to $3H^2/8\pi G$ turns out to be of the order of 10^{-123}. On the theoretical ground, such a fine tuning related to the scale of cosmological constant is not acceptable. This problem led to the investigation of scalar field models of dark energy which can alleviate this problem to a considerable extent.

1.3 Dynamically evolving scalar field models of dark energy

Before entering into the detailed investigations of field dynamics, we shall first examine some of the general constraints on scalar field Lagrangian if it is to be relevant to cosmology.

1.3.1 Broad features of scalar field dynamics and cosmological relevance of scaling solutions

The scalar field aiming to describe dark energy is often imagined to be a relic of early universe physics. Depending upon the model, the scalar field energy density may be larger or smaller than the background (radiation/matter) energy density ρ_B. In case it is larger than the back ground density, the density ρ_ϕ should scale faster than ρ_B allowing radiation domination to commence which requires a steep scalar field potential. In this case the field energy density overshoots the background and becomes sub dominant to it. This leads to the locking regime for the scalar field. The field unlocks the moment its energy density becomes comparable to the background. Its further course of evolution crucially depend upon the form of field potential. In order to obtain viable dark energy models, we require that the energy density of the scalar field remains unimportant during radiation and matter dominant eras and emerges only at late times to give rise to the current acceleration of universe. It is then important to investigate cosmological scenarios in which the energy density of the scalar field mimics the background energy density. The cosmological solutions which satisfy this condition is called *scaling solutions* [23]. Namely scaling solutions are characterised by the relation

$$\rho_B/\rho_\phi = \text{const}\,. \tag{1.92}$$

We shall shortly demonstrate that exponential potentials give rise to scaling solutions for a minimally coupled scalar field, allowing the field energy density to mimic the background being sub-dominant during radiation and matter dominant eras. In this case, for any generic initial conditions, the field would sooner or later enter into the scaling regime (see Fig.1.4). This allows to alleviate the fine tuning problem to a considerable extent. The same thing is true in case of the undershoot, i.e., when the field energy is smaller as compared to the background. In Fig.1.5, we have displayed a cartoon depicting the field dynamics in absence of scaling solutions. For instance, we shall see later, scaling solutions , which could mimic realistic background, do not exist in case of phantom and tachyon fields. These models are plagued with additional fine tuning problem.

Scaling solutions exist in case of a steep exponential potential $V(\phi) \sim exp(\lambda\phi/M_p)$ with $\lambda^2 > 3(1+w_m)$ (the field dominated case corresponds to $\lambda^2 < 3(1+w_m)$ whereas $\lambda^2 < 2$ gives rise to ever accelerating universe). Nucleosynthesis puts stringent restriction on any additional degree of freedom which translates into a constraint on the slope of the exponential potential λ.

• **Late time evolution and exit from scaling regime**

Obviously, scaling solution is non-accelerating as the equation of state of the field ϕ equals to that of the background fluid ($w_\phi = w_m$) in this case. One then requires to introduce a late time feature in the potential allowing to exit from the scaling regime. Broadly there are two ways to get the required late time behavior for a minimally coupled scalar field:
(i) The potential changes into a power law type $V \sim \phi^{2q}$ which gives late time acceleration for $q < 1/2$ (e.g., $V(\phi) = V_0 \left[\cosh(\alpha\phi/M_p) - 1\right]^q\,,\ q > 0$

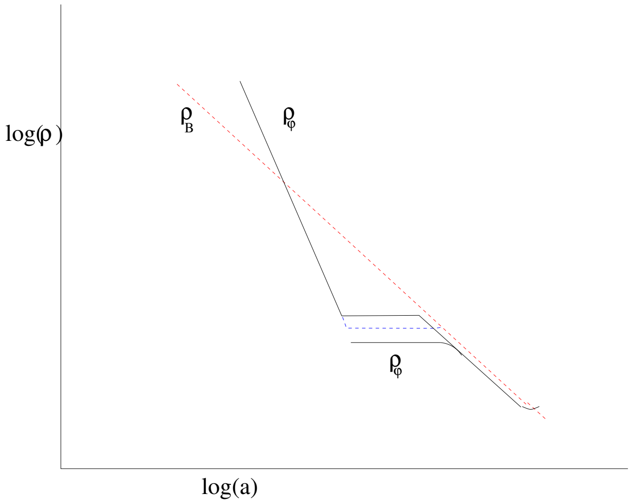

Fig. 1.4. Desired evolution of background and scalar field energy densities ρ_B and ρ_ϕ. In case of overshoot (solid line) and undershoot(dotted line), the field energy density (for different initial conditions) joins the attractor solution which mimics the background (*scaling solution*). At late times, the field energy density exits the scaling regime to become dominant.

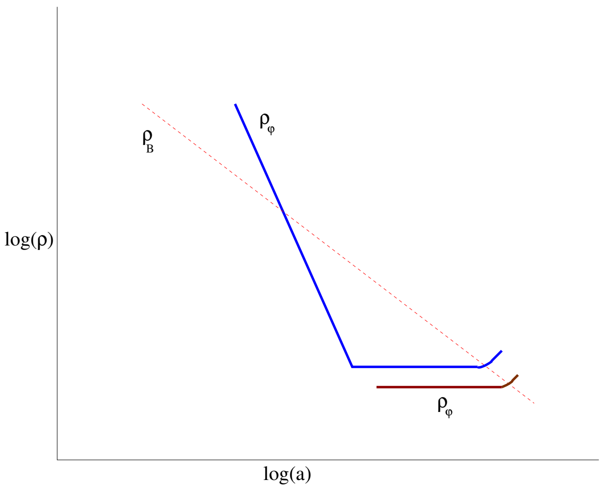

Fig. 1.5. Evolution of ρ_B, ρ_ϕ in absence of scaling solution. The scalar field after its energy density overshoots the background gets into locking regime where it mimics cosmological constant. It waits till its energy density becomes comparable to the background; it then begins evolving and takes over the background to account for the current acceleration

[24]).
(ii) The potential becomes shallow to support the slow-roll at large values of the field [25] allowing the field energy density to catch up with the background; such a solution is referred to a *tracker*.

The scalar field models in absence of the above described features suffer from the fine tuning problem similar to the case of cosmological constant.

Scalar fields should not interfere with the thermal history of universe, they are thus should satisfy certain constraints. An earlier constraint in the history of universe follows from nucleosynthesis which we briefly describe below[11].

• **Nucleosynthesis constraint**
The introduction of an extra degree of freedom (on the top of those already present in the standard model of particle physics) like a scalar field might effect the abundance of light elements in the radiation dominated epoch. The presence of a minimally coupled scalar field effects the expansion rate at a given temperature. This effect becomes crucial at the nucleosynthesis epoch with temperature round $1\,MeV$ when the weak interactions (which keep neutrons and protons in equilibrium) freeze-out. The observationally allowed range of expansion rate at this temperature leads to a bound on the energy density of the scalar field

$$\Omega_\phi(T \sim 1MeV) < \frac{7\Delta N_{eff}/4}{10.75 + 7\Delta N_{eff}/4} \,, \tag{1.93}$$

where ΔN_{eff} are the additional relativistic degrees of freedom and 10.75 is the effective number of standard model degrees of freedom. A conservative bound on the additional degrees of freedom used in the literature is given by $\Delta N_{eff} \simeq 1.5$. Equation (1.93) then yields a constraint

$$\Omega_\phi(T \sim 1MeV) < 0.2 \,, \tag{1.94}$$

which results into a restriction on the slope of the potential (see section V).

1.3.2 Autonomous systems, their fixed points and stability

The dynamical systems which play an important role in cosmology belong to the class of the so called autonomous systems. In what follows we shall analyze the dynamics in great details of a variety of scalar field models. We first briefly record some basic definitions related to dynamical systems. Though, for simplicity we shall consider the system of two first order equations, the analysis can be extended to a system of any number of equations. Let us consider the system of two coupled differential equations for $x(t)$ and $y(t)$

$$\begin{aligned} \dot{x} &= f(x, y, t) \,, \\ \dot{y} &= g(x, y, t) \,, \end{aligned} \tag{1.95}$$

where f and g are well behaved functions. System (1.95) is said to be autonomous if f and g do not contain explicit time dependent. The dynamics

of these systems can be analysed in a standard way.
• Fixed or critical points
A point (x_c, y_c) is said to be a *fixed point* or *critical point* of the autonomous system if and only if

$$(f, g)|_{x_c, y_c} = 0 \tag{1.96}$$

and a critical point (x_c, y_c) is called an *attractor* in case

$$(x(t), y(t)) \to (x_c, y_c) \ for \ t \to \infty \ . \tag{1.97}$$

• Stability around the fixed points
The stability of each point can be studied by considering small perturbations δx and δy around the critical point (x_c, y_c), i.e.,

$$x = x_c + \delta x \,, \quad y = y_c + \delta y \,. \tag{1.98}$$

Substituting into Eqs. (1.104) and (1.105), leads to the first-order differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}N} \begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = \mathcal{M} \begin{pmatrix} \delta x \\ \delta y \end{pmatrix} \,, \tag{1.99}$$

where matrix $\mathcal{M}$ depends upon x_c and y_c $\left[\mathcal{M} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix}_{(x=x_c, y=y_c)}\right]$

The general solution for the evolution of linear perturbations can be written as

$$\delta x = C_1 exp(\mu_1 N) + C_2 exp(\mu_2 N) \,, \tag{1.100}$$
$$\delta y = C_3 exp(\mu_1 N) + C_4 expp(\mu_2 N) \ , \tag{1.101}$$

where μ_1 and μ_2 are the eigenvalues of matrix $\mathcal{M}$. Thus the stability around the fixed points depends upon the nature of eigenvalues. One generally uses the following classification:

- (i) Stable node: $\mu_1 < 0$ and $\mu_2 < 0$.
- (ii) Unstable node: $\mu_1 > 0$ and $\mu_2 > 0$.
- (iii) Saddle point: $\mu_1 < 0$ and $\mu_2 > 0$ (or $\mu_1 > 0$ and $\mu_2 < 0$).
- (iv) Stable spiral: The determinant of the matrix $\mathcal{M}$ is negative and the real parts of μ_1 and μ_2 are negative.

1.3.3 Quintessence

Let us consider a minimally coupled scalar field ϕ with a potential $V(\phi)$:

$$\mathcal{L} = \frac{1}{2}\epsilon\dot{\phi}^2 + V(\phi) \,, \tag{1.102}$$

where $\epsilon = +1$ for an ordinary scalar field. Here we allow the possibility of phantom ($\epsilon = -1$) as we see in the next subsection.

In what follows we shall consider a cosmological evolution when the universe is filled by a scalar field ϕ and a barotropic fluid with an equation of state $w_m = p_m/\rho_m$. We introduce the following dimensionless quantities:

$$x \equiv \frac{\kappa\dot{\phi}}{\sqrt{6}H}\,, \quad y \equiv \frac{\kappa\sqrt{V}}{\sqrt{3}H}\,, \quad \lambda \equiv -\frac{V_\phi}{\kappa V}\,, \quad \Gamma = \frac{VV_{\phi\phi}}{V_\phi^2}\,. \tag{1.103}$$

For the Lagrangian density (1.102) the Einstein equations can be written in the following autonomous form (see Ref.[3] for details) :

$$\frac{\mathrm{d}x}{\mathrm{d}N} = -3x + \frac{\sqrt{6}}{2}\epsilon\lambda y^2 + \frac{3}{2}x\left[(1-w_m)\epsilon x^2 + (1+w_m)(1-y^2)\right]\,, \tag{1.104}$$

$$\frac{\mathrm{d}y}{\mathrm{d}N} = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y\left[(1-w_m)\epsilon x^2 + (1+w_m)(1-y^2)\right]\,, \tag{1.105}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}N} = -\sqrt{6}\lambda^2(\Gamma-1)x\,, \tag{1.106}$$

together with a constraint equation

$$\epsilon x^2 + y^2 + \frac{\kappa^2\rho_m}{3H^2} = 1\,, \tag{1.107}$$

where $N \equiv \log(a)$. We note that the equation of state w and the fraction of the energy density Ω_ϕ for the field ϕ is

$$w_\phi \equiv \frac{p}{\rho} = \frac{\epsilon x^2 - y^2}{\epsilon x^2 + y^2}\,, \quad \Omega_\phi \equiv \frac{\kappa^2\rho}{3H^2} = \epsilon x^2 + y^2\,. \tag{1.108}$$

We also define the total effective equation of state:

$$w_{\mathrm{eff}} \equiv \frac{p+p_m}{\rho+\rho_m} = w_m + (1-w_m)\epsilon x^2 - (1+w_m)y^2\,. \tag{1.109}$$

An accelerated expansion occurs for $w_{\mathrm{eff}} < -1/3$. In this subsection we shall consider the normal scalar field ($\epsilon = +1$).

Constant λ

From Eq. (1.103) we find that the constant λ corresponds to an exponential potential [23]:

$$V(\phi) = V_0 e^{-\kappa\lambda\phi}\,. \tag{1.110}$$

Name	x	y	Range	Stability	Ω_ϕ	γ_ϕ
(a)	0	0	$\forall\lambda,\ \gamma$	s. p. for $0 < \gamma < 2$	0	–
(b1)	1	0	$\forall\lambda,\gamma$	un. n. for $\lambda < \sqrt{6}$ s p for $\lambda > \sqrt{6}$	1	2
(b2)	-1	0	$\forall\lambda,\ \gamma$	un. n. for $\lambda > -\sqrt{6}$ s. p. for $\lambda < -\sqrt{6}$	1	2
(c)	$\lambda/\sqrt{6}$	$[1-\lambda^2/6]^{1/2}$	$\lambda^2 < 6$	st. n. for $\lambda^2 < 3\gamma$ st. n. for $3\gamma < \lambda^2 < 6$	1	$\lambda^2/3$
(d)	$(3/2)^{1/2}\,\gamma/\lambda$	$[3(2-\gamma)\gamma/2\lambda^2]^{1/2}$	$\lambda^2 > 3\gamma$	st. n. for $3\gamma < \lambda^2$ $< 24\gamma^2/(9\gamma-2)$ st. sp. for $\lambda^2 >$ $24\gamma^2/(9\gamma-2)$	$3\gamma/\lambda^2$	γ

Table 1.1. The properties of the critical points (s=saddle, p=point, un=unstable, n=node, st=stable, sp=spiral) from Ref.[3]. Here γ is defined by $\gamma \equiv 1 + w_m$.

In this case Eq. (1.106) is dropped from the dynamical system. One can obtain the fixed points by setting $\mathrm{d}x/\mathrm{d}N = 0$ and $\mathrm{d}y/\mathrm{d}N = 0$ in Eqs. (1.104) and (1.105). This is summarized in Table I.

In the next section we shall extend our analysis to the more general case in which dark energy is coupled to dark matter. The readers may refer to the next section in order to know precise values of the eigenvalues in a more general system. From TABLE I we find that there exists two stable fixed points (c) and (d). The point (c) is a stable node for $\lambda^2 < 3\gamma$. Since the effective equation of state is $w_{\rm eff} = w_\phi = -1+\lambda^2/3$, the accelerated expansion occurs for $\lambda^2 < 2$ in this case. The point (d) corresponds to a scaling solution in which the energy density of the field ϕ decreases proportionally to that of the barotropic fluid ($\gamma_\phi = \gamma$). Although this fixed point is stable for $\lambda^2 > 3\gamma$, we do not have an accelerated expansion in the case of non relativistic dark matter.

The above analysis of the critical points shows that one can obtain an accelerated expansion provided that the solutions approach the fixed point (c) with $\lambda^2 < 2$, in which case the final state of the universe is the scalar-field dominated one ($\Omega_\phi = 1$). The scaling solution (d) is not viable to explain the late-time acceleration. However this can be used to provide the cosmological evolution in which the scalar field decreases proportionally to that of the matter or radiation. If the slope of the exponential potential becomes shallow to satisfy $\lambda^2 < 2$ near to the present, the universe exits from the scaling regime and approaches the fixed point (c) giving rise to an accelerated expansion.

Dynamically changing $\boldsymbol{\lambda}$

Exponential potentials correspond to constant λ and $\Gamma = 1$. Let us consider the potential $V(\phi)$ along which the field rolls down toward plus infinity ($\phi \to$

∞) This means that $x > 0$ in Eq. (1.106). If the condition

$$\Gamma > 1\,, \tag{1.111}$$

is satisfied, λ decreases toward 0. Hence the slope of the potential becomes flat as $\lambda \to 0$, thereby giving rise to an accelerated expansion at late times. The condition (1.111) is regarded as the *tracking* condition under which the energy density of ϕ eventually catches up that of the fluid. In order to construct viable quintessence models, we require that the potential should satisfy the condition (1.111). For example, one has $\Gamma = (n+1)/n > 1$ for the inverse power-law potential $V(\phi) = V_0\phi^{-n}$ with $n > 0$. This means that the tracking occurs for this potential.

When $\Gamma < 1$ the quantity λ increases toward infinity. Since the potential is steep in this case, the energy density of the scalar field becomes negligible compared to that of the fluid. Hence we do not have an accelerated expansion at late times

In order to obtain the dynamical evolution of the system we need to solve Eq. (1.106) together with Eqs. (1.104) and (1.105). Although λ is dynamically changing, one can exploit the discussion of constant λ by considering "instantaneous" critical points.

1.3.4 Phantoms

The phantom field corresponds to a negative kinematic sign, i.e, $\epsilon = -1$ in Eq. (1.102). Let us consider the exponential potential given by Eq. (1.110). In this case Eq. (1.106) is dropped from the dynamical system. In Table 1.2 we show fixed points for the phantom field. The only stable solution is the scalar-field dominant solution (b), in which case the equation of the field ϕ is

$$w_\phi = -1 - \lambda^2/3\,. \tag{1.112}$$

Hence w_ϕ is less than -1. The scaling solution (c) is unstable and exists only for $w_m < -1$. We note that the effective equation of state of the universe equals to w_ϕ, i.e., $w_{\rm eff} = -1 - \lambda^2/3$. In this case the Hubble rate evolves as

$$H = \frac{2}{3(1+w_{\rm eff})(t-t_s)}\,, \tag{1.113}$$

where t_s is an integration constant. Hence H diverges for $t \to t_s$. This is so-called the Big Rip singularity at which the Hubble rate and the energy density of the universe exhibit divergence. We note that the phantom field rolls *up* the potential hill in order to lead to the increase of the energy density.

When the potential of the phantom is different from the exponential type, the quantity λ is dynamically changing in time. In this case the point (b) in Table 1.2 can be regarded as an instantaneous critical point. Then the equation of state w_ϕ varies in time, but the field behaves as a phantom since $w_\phi = -1 - \lambda^2/3 < -1$ is satisfied.

Name	x	y	Range	Stab.	Ω_ϕ	w_ϕ
(a)	0	0	No for $0 \le \Omega_\phi \le 1$	s. p.	0	–
(b)	$-\lambda/\sqrt{6}$	$[1+\lambda^2/6]^{1/2}$	All values	st. n.	1	$-1-\lambda^2/3$
(c)	$\frac{\sqrt{6}(1+w_m)}{2\lambda}$	$[\frac{-3(1-w_m^2)}{2\lambda^2}]^{1/2}$	$w_m < -1$	s. p.	$\frac{-3(1+w_m)}{\lambda^2}$	w_m

Table 1.2. The properties of the critical points (s=saddle, p=point, n=node, st=stable) for $\epsilon = -1$ (from[3]).

1.3.5 Tachyons

We shall take into account the contribution of a barotropic perfect fluid with an equation of state $p_B = (\gamma - 1)\rho_B$. Then the background equations of motion are for rolling tachyon system are

$$\dot{H} = -\frac{\dot{\phi}^2 V(\phi)}{2M_p^2\sqrt{1-\dot{\phi}^2}} - \frac{\gamma}{2}\frac{\rho_B}{M_p^2}\,, \tag{1.114}$$

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_\phi}{V} = 0\,, \tag{1.115}$$

$$\dot{\rho}_B + 3\gamma H \rho_B = 0\,, \tag{1.116}$$

together with a constraint equation:

$$3M_p^2 H^2 = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} + \rho_B\,. \tag{1.117}$$

Defining the following dimensionless quantities:

$$x = \dot{\phi}\,, \quad y = \frac{\sqrt{V(\phi)}}{\sqrt{3}HM_p}\,, \tag{1.118}$$

we obtain the following autonomous equations

$$\frac{\mathrm{d}x}{\mathrm{d}N} = -(1-x^2)(3x - \sqrt{3}\lambda y)\,, \tag{1.119}$$

$$\frac{\mathrm{d}y}{\mathrm{d}N} = \frac{y}{2}\left(-\sqrt{3}\lambda xy - \frac{3(\gamma - x^2)y^2}{\sqrt{1-x^2}} + 3\gamma\right)\,, \tag{1.120}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}N} = -\sqrt{3}\lambda^2 xy(\Gamma - 3/2)\,. \tag{1.121}$$

where

$$\lambda = -\frac{M_p V_\phi}{V^{3/2}}\,, \quad \Gamma = \frac{V V_{\phi\phi}}{V_\phi^2}\,. \tag{1.122}$$

We note that the allowed range of x and y is $0 \le x^2 + y^4 \le 1$ from the requirement: $0 \le \Omega_\phi \le 1$. Hence both x and y are finite in the range $0 \le x^2 \le 1$ and $0 \le y \le 1$. The effective equation of state for the field ϕ is

$$\gamma_\phi = \frac{\rho_\phi + p_\phi}{\rho_\phi} = \dot{\phi}^2 \,, \tag{1.123}$$

which means that $\gamma_\phi \ge 0$. The condition for inflation corresponds to $\dot{\phi}^2 < 2/3$.

Constant λ

From Eq. (1.121) we find that λ is a constant for $\Gamma = 3/2$. This case corresponds to an inverse square potential (For details, see Ref. [3])

$$V(\phi) = M^2 \phi^{-2} \,. \tag{1.124}$$

The scalar-field dominated solution ($\Omega_\phi = 1$), in this case, corresponds to $\gamma_\phi = \lambda^2/3$ which can lead to an accelerated expansion for $\lambda^2 < 2$. No scaling solution which could mimic radiation or matter exist in this case (see Ref.[3]). Since λ is given by $\lambda = 2M_p/M$, the condition for an accelerated expansion gives a super-Planckian value of the mass scale, i.e., $M > \sqrt{2}M_p$. Such a large mass is problematic since this shows the breakdown of classical gravity. This problem can be alleviated for the inverse power-law potential $V(\phi) = M^{4-n}\phi^{-n}$, as we will see below.

Dynamically changing λ

When the potential is different from the inverse square potential given in Eq. (1.124), λ is a dynamically changing quantity. As we have seen in the subsection of quintessence, there are basically two cases: (i) λ evolves toward zero, or (ii) $|\lambda|$ increases toward infinity. The case (i) is regarded as the tracking solution in which the energy density of the scalar field eventually dominates over that of the fluid. This situation is realized when the potential satisfies the condition

$$\Gamma > 3/2 \,, \tag{1.125}$$

as can be seen from Eq. (1.121). The case (ii) corresponds to the case in which the energy density of the scalar field becomes negligible compared to the fluid.

As an example let us consider the inverse power-law potential given by

$$V(\phi) = M^{4-n}\phi^{-n} \,, \qquad n > 0 \,. \tag{1.126}$$

In this case one has $\Gamma = (n+1)/n$. Hence the scalar-field energy density dominates at late times for $n < 2$.

x	y	Ω_ϕ	$w_{\rm eff}$
$-\frac{\sqrt{6}Q}{3(1-w_m)}$	0	$\frac{2Q^2}{3(1-w_m)}$	1
1	0	1	1
-1	0	1	1
$\frac{\lambda}{\sqrt{6}}$	$[(1-\frac{\lambda^2}{6})]^{1/2}$	1	$-1+\frac{\lambda^2}{3}$
$\frac{\sqrt{6}(1+w_m)}{2(\lambda+Q)}$	$[\frac{2Q(\lambda+Q)+3\ (1-w_m^2)}{2(\lambda+Q)^2}]^{1/2}$	$\frac{Q(\lambda+Q)+3\ (1+w_m)}{(\lambda+Q)^2}$	$\frac{\lambda w_m-Q}{(\lambda+Q)}$

Table 1.3. $Q \neq 0$, from[3]

There exist a number of potentials that exhibit the behavior $|\lambda| \to \infty$ asymptotically. For example $V(\phi) = M^{4-n}\phi^{-n}$ with $n > 2$ and $V(\phi) = V_0 e^{-\mu\phi}$ with $\mu > 0$. In the latter case one has $\Gamma = 1$. In these cases, pressure less dust ia late time attractor where as the accelerated expansion can occur as a transient phenomenon. Extra fine tuning is needed in this case to obtain the current acceleration.

1.4 Scaling solutions in models of coupled quintessence

As we have already seen in the previous section, exponential potentials give rise to scaling solutions for a minimally coupled scalar field, allowing the field energy density to mimic the background being sub-dominant during radiation and matter dominant eras. In the previous section we found out the expression for Ω_ϕ for scaling solution which after combining with the nucleosynthesis constraint (1.94) gives

$$\Omega_\phi \equiv \frac{\rho_\phi}{\rho_\phi + \rho_m} = \frac{(1+w_m)}{\lambda^2} < 0.2 \quad \to \quad \lambda > 5\ . \tag{1.127}$$

In this case, however, one can not have an accelerated expansion at late times since ρ_ϕ mimics background. We briefly mentioned as how to exit the scaling regime, in models of minimally coupled scalar fields, to account for the current acceleration of universe.

If the scalar field ϕ is coupled to the background fluid, it is possible to obtain an accelerated expansion at late-times even in the case of steep exponential potentials. In this section we implement the coupling Q between the field and the barotropic fluid and show that scaling solutions can also account for accelerated expansion. The evolution equations in presence of coupling acquire the form

$$\dot{\rho_\phi} + 3H(1+w_\phi)\rho_\phi = -Q\rho_m\dot{\phi} \tag{1.128}$$

$$\dot{\rho}_m + 3H(1+w_m)\rho_m = Q\rho_m\dot{\phi}\,, \tag{1.129}$$

$$\dot{H} = -\frac{1}{2}\Big[(1+w_m)\rho_\phi + (1+w_m)\rho_m\Big]\ . \tag{1.130}$$

$$H^2 = \frac{\rho_\phi + \rho_m}{3}\,, \tag{1.131}$$

where coupling Q is field dependent in general. For simplicity, we shall assume constant coupling. The autonomous form of equations for exponential potential in presence of coupling takes the following form

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3}{2}x\Big[(1-w_m)\epsilon x^2 + (1+w_m)(1-c_1 y^2)\Big] - \frac{\sqrt{6}Q}{2}(1-x^2-y^2)\,, \tag{1.132}$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y\left[(1-w_m)x^2 + (1+w_m)(1-y^2)\right]\,. \tag{1.133}$$

We display the critical points for coupled quintessence in the table in which the last entry corresponds to scaling solution with effective equation of state $w_{eff} = 0$ for $Q = 0$ consistent with earlier analysis. It is remarkable that $w_{eff} \to -1$ for $Q >> \lambda$. Thus scaling solutions can account for acceleration in presence of coupling between field and the barotropic fluid. Unfortunately, they are not acceptable from CMB constraints. The general investigations of perturbations for coupled quintessence require further serious considerations.

1.5 Quintessential inflation

In this section we shall work out the example of quintessential inflation which is an attempt to describe inflation and dark energy with a single scalar field. The description to follow would clearly demonstrate the utility of the tools developed in earlier sections. The problem was first addressed by Peebles and Vilenkin [26]. They introduced a potential for the field ϕ which allowed it to play the role of the inflaton in the early Universe and later to play the role of the quintessence field. To do this it was important that the potential did not have a minimum in which the inflaton field would completely decay at the end of the initial period of inflation. They proposed the following potential

$$V(\phi) = \begin{cases} \lambda(\phi^4 + M^4) & \text{for } \phi < 0\,, \\ \frac{\lambda M^4}{1+(\phi/M)^\alpha} & \text{for } \phi \geq 0\,. \end{cases} \tag{1.134}$$

For $\phi < 0$ we have ordinary chaotic inflation. Much later on, for $\phi > 0$ the universe once again begins to inflate but this time at the lower energy scale associated with quintessence. Reheating after inflation should have proceeded via gravitational particle production because of the absence of the potential minimum, but this mechanism is very inefficient and leads to an unwanted relic gravity wave background. The main difficulty for the realistic construction of quintessential inflation is that we need a flat potential during inflation but also require a steep potential during radiation and matter dominated periods. There are some nice resolutions of quintessential inflation in braneworld scenarios as we shall see below (see review.[27] and references therein on this theme). In these models, the scalar field exhibits the properties of tracker field. As a result it goes into hiding after the commencement of radiation domination; it emerges from the shadow only at late times to account for the observed accelerated expansion of universe. These models belong to the category of *non oscillating* models in which the standard reheating mechanism does not work. In this case, one can employ an alternative mechanism of reheating via quantum-mechanical particle production in time varying gravitational field at the end of inflation. However, then the inflaton energy density should red-shift faster than that of the produced particles so that radiation domination could commence. And this requires a steep field potential, which of course, cannot support inflation in the standard FRW cosmology. This is precisely where the brane[29] assisted inflation comes to our rescue. In the 4+1 dimensional brane scenario inspired by the Randall-Sundrum (RS) model, the standard Friedman equation is modified to

$$H^2 = \frac{1}{3M_p^2}\rho\left(1+\frac{\rho}{2\lambda_b}\right) \ , \tag{1.135}$$

The presence of the quadratic density term ρ^2/λ_b (high energy corrections) in the Friedmann equation on the brane changes the expansion dynamics at early epochs (see Ref[29] for details on the dynamics of brane worlds) Consequently, the field experiences greater damping and rolls down its potential slower than it would during the conventional inflation. This effect is reflected in the slow-roll parameters which have the form [29]

$$\begin{aligned} \epsilon &= \epsilon_{\rm FRW}\frac{1+V/\lambda_b}{\left(1+V/2\lambda_b\right)^2} \ , \\ \eta &= \eta_{\rm FRW}\left(1+V/2\lambda_b\right)^{-1} \ , \end{aligned} \tag{1.136}$$

where

$$\epsilon_{\rm FRW} = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta_{\rm FRW} = M_p^2\left(\frac{V''}{V}\right) \tag{1.137}$$

are slow roll parameters in the absence of brane corrections. The influence of the brane term becomes important when $V/\lambda_b \gg 1$ and in this case we get

$$\epsilon \simeq 4\epsilon_{\rm FRW}(V/\lambda_b)^{-1}, \, \eta \simeq 2\eta_{\rm FRW}(V/\lambda_b)^{-1}. \tag{1.138}$$

Clearly slow-roll ($\epsilon, \eta \ll 1$) is easier to achieve when $V/\lambda_b \gg 1$ and on this basis one can expect inflation to occur even for relatively steep potentials, such the exponential and the inverse power-law. The model of quintessential inflation [27] based upon reheating via gravitational particle production is faced with difficulties associated with excessive production of gravity waves.

Indeed the reheating mechanism based upon this process is extremely inefficient. The energy density of so produced radiation sis typically one part in 10^{16} to the scalar-field energy density at the end of inflation. As a result, these models have prolonged kinetic regime during which the amplitude of primordial gravity waves enhances and violates the nucleosynthesis constraint. Hence, it is necessary to look for alternative mechanisms more efficient than the gravitational particle production to address the problem. However this problem may be alleviated in instant preheating scenario [28] in the presence of an interaction $g^2\phi^2\chi^2$ between inflaton ϕ and another field χ. This mechanism is quite efficient and robust, and is well suited to non-oscillating models. It describes a new method of realizing quintessential inflation on the brane in which inflation is followed by 'instant preheating'. The larger reheating temperature in this model results in a smaller amplitude of relic gravity waves which is consistent with the nucleosynthesis bounds[27]. Fig.1.6 shows the post inflationary evolution of scalar field energy density for the potential given by

$$V(\phi) = V_0\left[\cosh(\kappa\lambda\phi) - 1\right]^n \ . \tag{1.139}$$

This potential has following asymptotic forms:

$$V(\phi) \simeq \begin{cases} \widetilde{V}_0 e^{-n\kappa\lambda\phi} & (|\lambda\phi| \gg 1,\ \phi < 0)\,, \\ \widetilde{V}_0(\kappa\lambda\phi)^{2n} & (|\lambda\phi| \ll 1)\,, \end{cases} \tag{1.140}$$

where $\widetilde{V}_0 = V_0/2^n$. The existence of scaling solution for exponential potential ($V \sim exp(\kappa\lambda\phi)$) tells us that $\lambda^2 > 3\gamma$ where as nucleosynthesis constraint makes the potential further steeper as $\Omega_\phi = 3\gamma/\lambda^2 < 0.2 \ \rightarrow \ \lambda > 5$. Potential (1.140) is suitable for unification of inflation and quintessence. In this case, for a given number of e-foldings, the COBE normalization allows to estimate the brane tension λ_b and the field potential at the end of inflation. Tuning the model parameters (λ − slope of the potential and V_0), we can account for the current acceleration with $\Omega_\phi^{(0)} \simeq 0.7$ and $\Omega_m^{(0)} \simeq 0.3$[27]. However, the recent measurement of CMB anisotropies by WMAP places fairly strong constraints on inflationary models. The ratio of tensor perturbations to scalar perturbations turns out to be large in case of steep exponential potential pushing the model outside the 2σ observational bound [30]. However, the model can be rescued in case a Gauss-Bonnet term is present in five dimensional bulk [31, 32]. In order to see how it comes about, let us consider Einstein-Gauss-Bonnet action for five dimensional bulk containing a 4D brane

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\{R - 2\Lambda_5 + \alpha_{\rm GB}[R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}]\} + \int d^4x\sqrt{-h}(L_m - \lambda_b)\ , \tag{1.141}$$

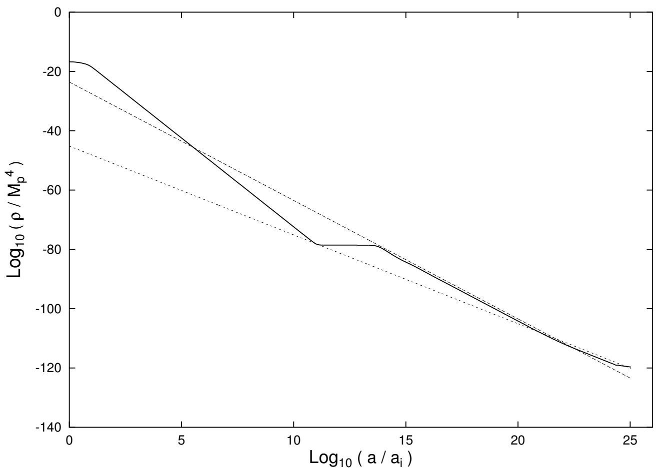

Fig. 1.6. The post-inflationary evolution of the scalar field energy density (solid line), radiation (dashed line) and cold dark matter (dotted line) is shown as a function of the scale factor for the quintessential inflation model described by (1.140) with $V_0^{1/4} \simeq 10^{-30} M_p$, $\lambda = 50$ and $n = 0.1$. After brane effects have ended, the field energy density ρ_ϕ enters the kinetic regime and soon drops below the radiation density. After a brief interval during which $< w_\phi > \simeq -1$, the scalar field begins to track first radiation and then matter. At very late times (present epoch) the scalar field plays the role of quintessence and makes the universe accelerate. The evolution of the energy density is shown from the end of inflation until the present epoch. From Ref.[32].

R refers to the Ricci scalars in the bulk metric g_{AB} and h_{AB} is the induced metric on the brane; $\alpha_{\rm GB}$ has dimensions of $(length)^2$ and is the Gauss-Bonnet coupling, while λ_b is the brane tension and Λ_5 (< 0) is the bulk cosmological constant. The constant κ_5 contains the M_5, the 5D fundamental energy scale $(\kappa_5^2 = M_5^{-3})$.

The analysis of modified Friedmann [33] equation which follows from the above action shows that there is a characteristic GB energy scale $M_{\rm GB}$[33] such that,

$$\rho \gg M_{\rm GB}^4 \;\Rightarrow\; H^2 \approx \left[\frac{\kappa_5^2}{16\alpha_{\rm GB}}\,\rho\right]^{2/3}, \tag{1.142}$$

$$M_{\rm GB}^4 \gg \rho \gg \lambda_b \;\Rightarrow\; H^2 \approx \frac{\kappa^2}{6\lambda_b}\,\rho^2\,, \tag{1.143}$$

$$\rho \ll \lambda_b \;\Rightarrow\; H^2 \approx \frac{\kappa^2}{3}\,\rho\,. \tag{1.144}$$

It should be noted that Hubble law acquires an unusual form for energies higher than the GB scale. Interestingly, for an exponential potential, the modified Eq.(1.142) leads to exactly scale invariant spectrum for primordial density perturbations. Inflation continues below GB scale and terminates in the RS regime leading to the spectral index very close to one. However, as shown in Refs.[33, 31], the tensor to scalar ratio of perturbations(R) also increases towards the high energy GB regime. It is known that the value of R is larger in case of RS brane world as compared to the standard GR. While moving from the RS regime characterized by $H^2 \propto \rho^2$ to GB regime described by $H^2 \propto \rho^{2/3}$, we pass through an intermediate region which mimics GR like behavior. It is not surprising that the ratio R has minimum at an intermediate energy scale between RS and GB, see Fig.1.7. We conclude that a successful scenario of quintessential inflation on the Gauss-Bonnet braneworld can be constructed which agrees with CMB+LSS observations.

1.6 Conclusions

In this talk we have reviewed the general features of scalar field dynamics. Our discussion has been mainly pedagogical in nature. we tried to present the basic features of standard scalar field, phantoms and rolling tachyon. Introducing the basic definitions and concepts, we have shown as how to find the critical pints and investigate stability around them. This is a standard technique needed for building the scalar field models desired for a viable cosmic evolution. The two often used mechanisms for the exit from scaling regime are also described in detail. In case of phantoms and rolling tachyon, we have shown that there exits no scaling solutions which would mimic the realistic background fluid (radiation/matter). Thus, in these case, there will be dependency on the initial conditions of the field leading to fine tuning problems. These models should therefore be judged on the basis of generic features which might arise in them. The rolling tachyon is inspired by string theory whereas as phantoms might be supported by observations!.

After developing the basic techniques of scalar field dynamics, we worked out the example of quintessential inflation. we have shown in detail how to implement the techniques for building a unified model of inflation and quintessence with a single scalar field.

In this talk we have not touched upon the observational status of dark energy models. We have also not discussed the alternatives to dark energy. The interested reader is refereed to other talks on these topics in the same proceedings. The supernovae observations are not yet sufficient to decide the metamorphosis of dark energy. There have been claims and anti-claims for dynamically evolving dark energy using supernovae, CMB and large scale studies. Given the present observational status of cosmology, it would be

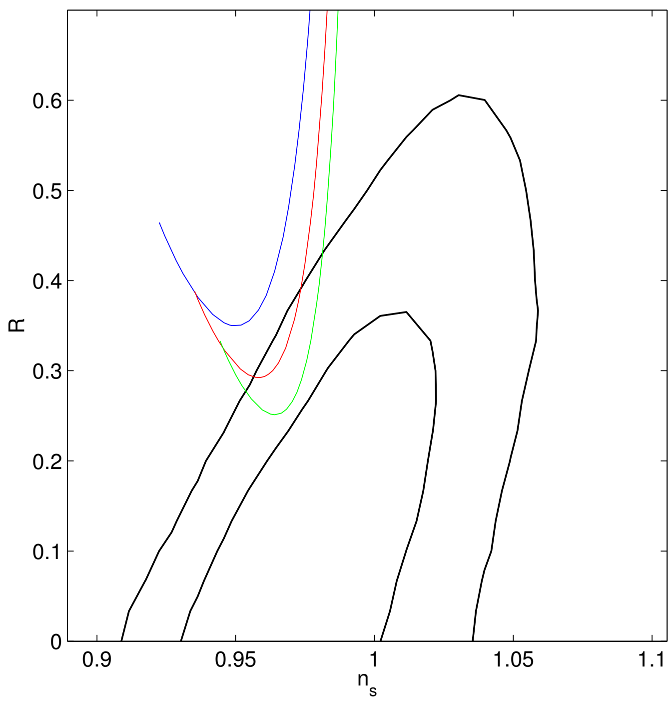

Fig. 1.7. Plot of R ($R \equiv 16A_T^2/A_S^2$ − according to the normalization used here[31]) versus the spectral index n_S in case of the exponential potential for the number of inflationary e-foldings $\mathcal{N} = 50, 60, 70$ (from top to bottom) along with the 1σ and 2σ observational contours. These curves exhibit a minimum in the intermediate region between GB (extreme right) and the RS (extreme left) regimes. The upper limit on n_S is dictated by the quantum gravity limit where as the lower bound is fixed by the requirement of ending inflation in the RS regime[31]. For a larger value of the number of e-folds $\mathcal{N}$, more points are seen to be within the 2σ bound. Clearly, steep inflation in the deep GB regime is not favored due to the large value of R in spite the spectral index being very close to 1 there. From Ref.[32]

fair to say that the nature of dark energy remains to be a mystery of the millennium. It could be any thing or it could be nothing!

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